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Dynamics for a class of non-autonomous degenerate p-Laplacian equations

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ABSTRACT

In this paper, we investigate a class of non-autonomous degenerate p-Laplacian equations

$$\partial_t u - \operatorname{div}(a(x)|\nabla u|^{p-2}\nabla u) + \lambda u + f(u) = g(x,t)$$

in Ω, where $a(x)$ is allowed to vanish on a nonempty closed subset with Lebesgue measure zero, $g(x,t) \in L^{p'}_{loc}(\mathbb{R};\ D^{-1,p'}(\Omega,a))$ and Ω an unbounded domain in $\mathbb{R}^N$. We first establish the well-posedness of these equations by constructing a compact embedding. Then we show the existence of the minimal pullback $\mathscr{D}_\mu$-attractor, and prove that it indeed attracts the $\mathscr{D}_\mu$ class in $L^{2+\delta}$-norm for any $\delta \in [0,\infty)$. Our results extend some known ones in previously published papers.

1. Introduction

In this article, we are concerned with the following non-autonomous degenerate p-Laplacian equation in an unbounded and smooth domain Ω in $\mathbb{R}^N (N > 2)$,

$$\begin{cases} \frac{\partial u}{\partial t} - \operatorname{div}(a(x)|\nabla u|^{p-2}\nabla u) + \lambda u + f(u) = g(x,t) & \text{in } \Omega \times (\tau,\infty), \\ u(x,t) = 0 & \text{on } \partial\Omega \times [\tau,\infty), \\ u(x,\tau) = u_\tau & \text{in } \Omega, \end{cases} \tag{1.1}$$

where $p \in (2,N)$, $u_\tau \in L^2(\Omega)$ and $\tau \in \mathbb{R}$ is the initial time. The variable diffusion coefficient $a(x)$ is a nonnegative and continuous function on the closure of Ω, which is denoted by $\overline{\Omega}$. Here the degeneracy

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means that $a(x)$ vanishes on some nonempty subsets of Ω. We assume that $a(x)$ satisfies the following two conditions.

$(\mathcal{H}_\alpha)$ There exist constants $r > 0$, $\alpha \in (0, p)$ and a subset $\Sigma \subset \Omega$ with properties $\Sigma \neq \emptyset$, $\Sigma = \overline{\Sigma}$ and $\mathcal{L}^N(\Sigma) = 0$, where $\overline{\Sigma}$ denotes the closure of Σ, $\mathcal{L}^N(\Sigma)$ denotes the Lebesgue measure of Σ on $\mathbb{R}^N$, such that

$$\begin{cases} a(x) = 0 & \text{if} \quad x \in \Sigma, \\ a(x) > 0 & \text{if} \quad x \in \overline{\Omega}/\overline{\Sigma}, \end{cases}$$

and

$$\int\limits_{\Omega \cap B_r(x_0)} \frac{1}{a(x)^{\frac{N}{\alpha}}} dx < +\infty, \quad \text{for any} \quad x_0 \in \Sigma; \tag{1.2}$$

$(\mathcal{H}_\beta^\infty)$ There exists $\beta \in (p + \frac{N}{2}(p-2), (p-1)N]$ such that $\liminf_{|x|\to\infty} |x|^{-\beta} a(x) > 0$.

The condition $(\mathcal{H}_\alpha)$ implies that the variable diffusion coefficient is allowed to vanish on a nonempty closed subset of Ω with N-dimensional Lebesgue measure zero, and the rate of degeneracy of $a(x)$ is measured by (1.2).

We assume that the nonlinear term $f \in C^1(\mathbb{R}, \mathbb{R})$ satisfies the following conditions:

$$\exists\, C_1, C_2, k_1, k_2 > 0,\ q \geqslant 2,\ k_1 < \lambda,\ \text{s.t.}\ C_1|s|^q - k_1|s|^2 \leqslant f(s)s \leqslant C_2|s|^q + k_2|s|^2,\ \forall\, s \in \mathbb{R}, \tag{1.3}$$

and

$$\exists\, l > 0,\ \text{s.t.}\ f'(s) \geqslant -l, \quad \forall\, s \in \mathbb{R}. \tag{1.4}$$

Suppose that the time-depending forcing term g satisfies

$$g \in L^{p'}_{loc}(\mathbb{R}; D^{-1,p'}(\Omega, a)), \tag{1.5}$$

where $D^{-1,p'}(\Omega, a)$ is the dual space of the weighted Sobolev space $D_0^{1,p}(\Omega, a)$ (see Section 2.1 for its definition), and p' is the conjugate exponent of p, i.e., $\frac{1}{p} + \frac{1}{p'} = 1$.

Evolution equations of type (1.1) occur in many fields: mechanics, physics, chemistry and materials science (non-Newtonian fluids, flows in porous media, electromagnetic phenomena in nonhomogeneous superconductors, etc.), see e.g. [5,6,11,15,17]. The physical meaning of the variable diffusion coefficients of these equations can be seen in the theory of superconductivity. Indeed, the coefficient represents the equilibrium density of superconducting electrons in a material. When the conduction of a material is inhomogeneous, the density can be considered as a function of position (see [12,18,19]).

In the study of nonhomogeneous superconductors, the degeneracy is related to materials which at some points behave as *perfect insulators*. Following Dautray and Lions' book [14], on the one hand, it is natural to assume that the coefficient $a(x)$ vanishes on some points when a medium is perfectly insulating at these points, and in this case $(\mathcal{H}_\alpha)$ means that a medium acts as perfect insulators on some "thin layers" in the interior of the medium. On the other hand, when a medium is perfectly conducting near the infinity, we can assume naturally that $a(x)$ is strictly monotone increasing with respect to the module of x, in which case the meaning of $(\mathcal{H}_\beta^\infty)$ is that a medium behaves as a perfect conductor.

It is worth mentioning that the first assumption on the diffusion coefficient is proposed by Caldiroli and Musina [8], which is formulated as follows: