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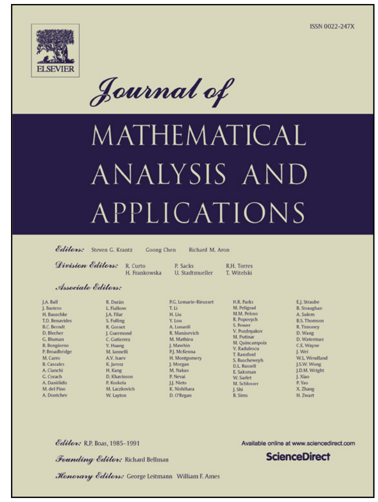

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Local convergence analysis of Newton's method for solving strongly regular generalized equations

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Abstract

In this paper, we consider Newton's method for solving a generalized equation of the form $f(x) + F(x) \ni 0$, where $f : \Omega \to Y$ is continuously differentiable, X and Y are Banach spaces, $\Omega \subset X$ is open, and $F : X \rightrightarrows Y$ has a nonempty closed graph. We show that, under strong regularity of the equation, the method is locally convergent to a solution with superlinear/quadratic rate. Our analysis, which is based on general majorant condition, enables us to obtain a convergence result under the Lipschitz, Smale's, and Nesterov-Nemirovskii's self-concordant conditions.

Keywords: Generalized equation, Newton's method, strong regularity, majorant condition.
Mathematical Subject Classification (2010): Primary 65K15; 49M15; 90C31

1 Introduction

In this paper, we consider Newton's method for solving a generalized equation of the form

$$f(x) + F(x) \ni 0, \tag{1}$$

where $f : \Omega \to Y$ is a continuously differentiable function, X and Y are Banach spaces, $\Omega \subset X$ is an open set and $F : X \rightrightarrows Y$ is a set-valued mapping with a closed nonempty graph. As is well-known, (1) is an abstract model for a wide range of problems in mathematical programming, and therefore, it has been studied in several works having [3, 4, 11, 15, 16, 17, 25, 26, 27, 28, 35] as part of a whole. For instance, if Y is the dual X^* of X and F is the normal cone mapping N_C of a closed convex set $C \subset X$, then (1) is called variational inequality for f and C; for more details, see [1, 3, 15].

Newton's method is undoubtedly one of the most popular methods for numerically solving nonlinear equations due to its property of quadratic convergence. Over the years, this method has been extended in many directions by several authors; one of the most studied currently is its generalization to solve (1), which has its origin in the works of N. H. Josephy [26]. Following the idea of [26], we study local convergence of the following Newton's method for solving (1): For an initial point x_0, define

$$f(x_k) + f'(x_k)(x_{k+1} - x_k) + F(x_{k+1}) \ni 0, \qquad k = 0, 1, \ldots. \tag{2}$$

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