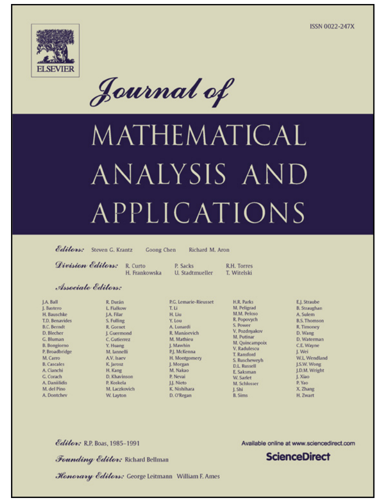

Large deviations for subordinated fractional Brownian motion and applications

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LARGE DEVIATIONS FOR SUBORDINATED FRACTIONAL BROWNIAN MOTION AND APPLICATIONS

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ABSTRACT. Let $W^H = \{W^H(t), t \in \mathbb{R}\}$ be a real valued fractional Brownian motion with Hurst index $H \in (0, 1)$ and let $T = \{T_t, t \geq 0\}$ be an inverse α-stable subordinator independent of W^H. The inverse stable subordintor fractional Brownian motion $Z^H = \{Z^H(t), t \geq 0\}$ is defined by $Z^H(t) = W^H(T_t)$, which may arise as scaling limit of CTRW or random walk in a random environment. In this paper we establish large deviation results for the process Z^H and its supremum process. And we also give asymptotic properties of the tail probability of the supremum process.

1. Introduction and statement of results

Fractional Brownian motion (fBm) is a centered Gaussian process $W^H = \{W^H(t), t \in \mathbb{R}\}$ with $W^H(0) = 0$ and covariance function

$$\mathbb{E}\left(W^H(s)W^H(t)\right) = \frac{1}{2}\left(|s|^{2H} + |t|^{2H} - |s-t|^{2H}\right),$$

where $H \in (0, 1)$ is a constant. It is known that W^H is self-similar with index H (i.e., for all constants $c > 0$, the processes $\{W^H(ct), t \in \mathbb{R}\}$ and $\{c^H W^H(t), t \in \mathbb{R}\}$ have the same finite-dimensional distributions) and has stationary increments. When $H = 1/2$, W^H is a two-sided Brownian motion, which will be written as W.

FBm is an improtant example of self-similar processes which arise naturally in limit theorems of random walks and other stochastic processes, and it has been applied to model various phenomena in a wide range of scientific ares including telecommunications, turbulence, image processing and finance.

In this paper, we consider a class of iterated self-similar processes which is related to continuous-time random walks considered in [3, 13]. Let $X = \{X_t, t \geq 0\}$ be a real-valued α-stable subordinator, where $0 < \alpha < 1$. We assume that X is independent of W^H, $T = \{T_t, t \geq 0\}$ be the inverse process of X, i.e $T_t = \inf\{\tau; X_\tau > t\}$. Let $Z^H = \{Z^H(t), t \geq 0\}$ be the real-valued stochastic process defined by $Z^H(t) = W^H(T_t)$ for all $t \geq 0$. This iterated process will be called subordinated fractional Brownian motion. When $H = 1/2$, $Z^H(t)$ will be written as Z, which is also called fractional kinetic process [2, 12].

References [8, 11] proved the large deviations for subordinated fractional Brownian motion under the condition of $2H(1-\alpha) < 1$. In this paper, we will establish large deviations for the subordinated fractional Brownian motion Z^H and supremum $\sup_{0 \leq s \leq t} Z^H(s)$ without the exact condition. And we also give asymptotic properties of the tail probability of the supremum process in Section 4. The following are our main results.

Theorem 1.1. *Let $Z^H = \{Z^H(t), t \geq 0\}$ be real-valued subordinated fractional Brownian motion. Then for every $\beta > 0$ such that $0 < \beta(\frac{1}{2} + H - \alpha H) < 1$, every function $a(t)$ with $\lim_{t \to \infty} a(t) = \infty$, and every Borel set $D \subseteq \mathbb{R}$,*

$$\limsup_{t \to \infty} \frac{1}{a(t)^{1/[1-\beta(\frac{1}{2}+H-\alpha H)]}} \log \mathbb{P}\left\{\frac{|Z^H(t)|^\beta}{a(t)^{\beta(\frac{1}{2}+H-\alpha H)/[1-\beta(\frac{1}{2}+H-\alpha H)]} t^{\alpha\beta H}} \in D\right\} \leq -\inf_{x \in \bar{D}} \Lambda_1^*(x) \quad (1)$$

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