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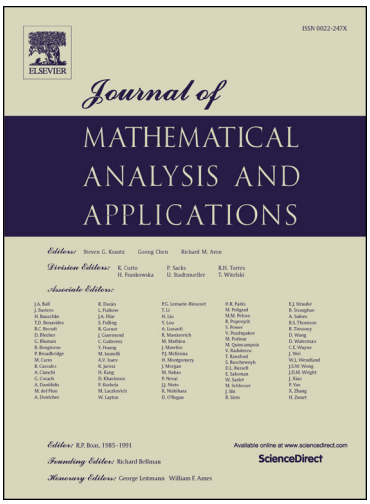

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A Remark on Denjoy's inequality for PL circle homeomorphisms with two break points [1]

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Abstract

It is well known that for a P-homeomorphism f of the circle $S^1 = \mathbb{R}/\mathbb{Z}$ with irrational rotation number ρ_f the Denjoy's inequality $|\log Df^{q_n}| \leq V$ holds, where V is the total variation of $\log Df$ and q_n, $n \geq 1$, are the first return times of f. Let h be a piecewise-linear (PL) circle homeomorphism with two break points a_0, c_0, irrational rotation number ρ_h and total jump ratio $\sigma_h = 1$. Denote by $\mathbf{B}_n(h)$ the partition determined by the break points of h^{q_n} and by μ_h the unique h-invariant probability measure. It is shown that the derivative Dh^{q_n} is constant on every element of $\mathbf{B}_n(h)$ and takes either two or three values. Furthermore we prove, that $\log Dh^{q_n}$ can be expressed in terms of μ_h- measures of some intervals of the partition $\mathbf{B}_n(h)$ multiplied by the logarithm of the jump ratio $\sigma_h(a_0)$ of h at the break point a_0.

1 Introduction

Let f be an orientation preserving homeomorphism of the circle $S^1 \equiv \mathbb{R}/\mathbb{Z}$ with lift $F : \mathbb{R} \to \mathbb{R}$, which is continuous, increasing and fulfills $F(\hat{x}+1) = F(\hat{x}) + 1$ for $\hat{x} \in \mathbb{R}$ a lift of x. The circle homeomorphism f is then defined by $f(x) = F(\hat{x}) \ (mod\, 1)$, $x \in S^1$. The **rotation number** ρ_f is defined by $\rho_f := \lim_{n\to\infty} \frac{F^n(\hat{x})-\hat{x}}{n} \ (mod\, 1)$. Here and below, F^i denotes the i-th iteration of the lift F. It is well known, that the rotation number ρ_f does not depend on the point $\hat{x} \in \mathbb{R}$ and is irrational if and only if f has no periodic points (see [5]). The rotation number ρ_f is invariant under topological conjugations. We shall assume the rotation number ρ_f to be irrational throughout this paper. We use the continued fraction representation $\rho_f = 1/(k_1 + 1/(k_2 + ...)) := [k_1, k_2, ..., k_n, ...)$ of the rotation number ρ_f. Denote by $p_n/q_n = [k_1, k_2, ..., k_n]$, $n \geq 1$, its $n-$th convergent. The numbers $q_n, n \geq 1$ are the **first return times** of f and satisfy the recursive relations $q_{n+1} = k_{n+1}q_n + q_{n-1}$ for $n \geq 1$, where $q_0 = 1$, and $q_1 = k_1$.

A natural extension of circle diffeomorphisms are piecewise smooth homeomorphisms with break points or shortly, the class of P-homeomorphisms.

The class of **P-homeomorphisms** consists of orientation preserving circle homeomorphisms f which are differentiable except at a finite or countable number of break points x_b, at which the one-sided positive derivatives Df_- and Df_+ exist, which do not coincide and for which there exist constants $0 < c_1 < c_2 < \infty$, such that

- $c_1 < Df_-(x_b) < c_2$ and $c_1 < Df_+(x_b) < c_2$ for all $x_b \in BP(f)$, the set of break points of f in S^1;
- $c_1 < Df(x) < c_2$ for all $x \in S^1 \backslash BP(f)$;
- $\log Df$ has finite total variation in S^1.

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