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Existence of a strong solution and trajectory attractor for a climate dynamics model with topography effects

Ruxu Lian [a,b,*], Qingcun Zeng [b]

[a] College of Mathematics and Information Science, North China University of Water Resources and Electric Power, Zhengzhou 450011, PR China
[b] Institute of Atmospheric Physics, Chinese Academy of Sciences, Beijing 100029, PR China

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ABSTRACT

The primitive three-dimensional viscous equations for large-scale atmosphere dynamics are commonly used in weather and climate predictions, and multiple theoretical analyses have been performed on them. However, few studies have considered topographic effects, which have a remarkable influence on climate factors (e.g., atmospheric temperature and wind velocity). In this study, a climate dynamics model with topography and non-stationary external force effects based on the Navier–Stokes equations and a temperature equation is analyzed. The existence and uniqueness of a global strong solution for this system is demonstrated based on the initial data assumptions. In addition, the existence of a universal attractor in the dynamic system is confirmed.

1. Introduction

The so-called primitive atmospheric equations were first derived by Richardson [20] and consisted of the hydrodynamic equations with the Coriolis force and the thermodynamic equations. However, the system was too complicated to be studied theoretically or to be solved numerically. By making the hydrostatic approximation, the primitive equations were formulated into the Navier–Stokes equations with the Coriolis force, the thermodynamic equations and the diffusion equation for vapor [16,19,24]. Most models have not discussed the effects of topography or changes of the external forcing with time. However, many observations have indicated that such environmental conditions play a vital role in climate dynamics. To describe realistic conditions, Zeng [25] modified the climate dynamics model mentioned in [16,19,24] in the following ways: (1) the effects of topography on the climate dynamics were considered; (2) non-stationary external forcing (e.g., diabatic atmospheric heating) was included; (3) the upper atmospheric pressure was set to zero rather than a small positive constant; and (4) the anelastic approximation was not used in the dynamic system.

* Corresponding author.
E-mail address: ruxu.lian.math@gmail.com (R. Lian).

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We introduce a moving frame $(\theta, \lambda, \zeta, t)$, where $\theta \in [0, \pi]$ is the colatitude, $\lambda \in [0, 2\pi]$ is the longitude, and $\zeta = p/p_s \in [0, 1]$, $p \in [0, p_s]$ is the atmospheric pressure, $p_s(\theta, \lambda, t)$ is the atmospheric pressure on the surface of the earth and t is the time. The atmospheric state functions can be defined by the atmospheric horizontal velocity $V = (v_\theta, v_\lambda)$, vertical velocity $\dot{\zeta}$, temperature deviation T', geopotential deviation Φ' and earth surface pressure deviation p'_s if the reference standard temperature $\widetilde{T}(\zeta)$, reference standard geopotential $\widetilde{\Phi}(\zeta)$ and reference standard earth surface pressure $\widetilde{p_s}(\theta, \lambda)$ are given. T' suggests that $\widetilde{T}(\zeta) + T'(\theta, \lambda, \zeta, t)$ is the atmospheric temperature $T(\theta, \lambda, \zeta, t)$, Φ' suggests that $\widetilde{\Phi}(\zeta) + \Phi'(\theta, \lambda, \zeta, t)$ is the geopotential $\Phi(\theta, \lambda, \zeta, t)$ and p'_s suggests that $\widetilde{p_s}(\theta, \lambda) + p'_s(\theta, \lambda, t)$ is the surface pressure of earth $p_s(\theta, \lambda, t)$. All of these conditions satisfy the following system

$$
\begin{cases}
\dfrac{\partial V}{\partial t} + (V^* \cdot \nabla) V + \dot{\zeta}^* \dfrac{\partial V}{\partial \zeta} + (2\omega \cos\theta + \dfrac{\cot\theta}{a} v_\lambda) \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} V + \nabla \Phi' + RT' \dfrac{\nabla \widetilde{p_s}}{\widetilde{p_s}} \\
= \dfrac{\mu_1}{\widetilde{p_s}} \Delta V + \nu_1 \dfrac{\partial}{\partial \zeta} \big((\dfrac{g\zeta}{R\widetilde{T}})^2 \dfrac{\partial V}{\partial \zeta} \big), \\
\dfrac{\partial T'}{\partial t} + (V^* \cdot \nabla) T' + \dot{\zeta}^* \dfrac{\partial T'}{\partial \zeta} - \dfrac{c_0^2}{\widetilde{p_s}\zeta} \big(\widetilde{p_s}\dot{\zeta} + \zeta (\dfrac{\partial p'_s}{\partial t} + \nabla \widetilde{p_s} \cdot V) \big) \\
= \dfrac{1}{c_p} \left(\dfrac{\mu_2}{\widetilde{p_s}} \Delta T' + \nu_2 \dfrac{\partial}{\partial \zeta} \big((\dfrac{g\zeta}{R\widetilde{T}})^2 \dfrac{\partial T'}{\partial \zeta} \big) \right) + \dfrac{\Psi}{c_p}, \\
\dfrac{\partial p'_s}{\partial t} + \nabla \cdot (\widetilde{p_s} V) + \dfrac{\partial \widetilde{p_s} \dot{\zeta}}{\partial \zeta} = 0, \\
\dfrac{\partial \Phi'}{\partial \zeta} + \dfrac{RT'}{\zeta} = 0,
\end{cases}
\tag{1.1}
$$

where ω is the angular velocity of the earth; g is the acceleration due to gravity; c_0, c_p and R are the thermodynamics parameters; μ_i and $\nu_i (i = 1, 2)$ are the diffusion coefficients; $2\omega \cos\theta \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} V$ denotes the Coriolis force on the atmosphere; Ψ is the diabatic atmospheric heating as a function of $(\theta, \lambda, \zeta, t)$ and stands for the effect of the non-constant external force on the atmospheric system.

The differential operators grad $:= \nabla$, div $:= \nabla \cdot$ and Δ on the spherical surface take the following forms

$$
\begin{cases}
\nabla = (\dfrac{1}{a} \dfrac{\partial}{\partial \theta}, \dfrac{1}{a \sin\theta} \dfrac{\partial}{\partial \lambda}), \\
\nabla \cdot V = \dfrac{1}{a \sin\theta} \dfrac{\partial (\sin\theta v_\theta)}{\partial \theta} + \dfrac{1}{a \sin\theta} \dfrac{\partial v_\lambda}{\partial \lambda}, \\
\Delta = \dfrac{1}{a^2 \sin\theta} \dfrac{\partial}{\partial \theta} \big(\sin\theta \dfrac{\partial}{\partial \theta} \big) + \dfrac{1}{a^2 \sin^2\theta} \dfrac{\partial^2}{\partial \lambda^2},
\end{cases}
\tag{1.2}
$$

where a is the radius of the earth. The vertical scale of the atmosphere is very smaller compared to the radius of the earth. Therefore, the geocentric distance r is replaced by the radius of the earth in the differential operators. The above equations are studied on $\Omega \times [0, M] := S^2 \times [0, 1] \times [0, M] = [0, \pi] \times [0, 2\pi] \times [0, 1] \times [0, M]$, where $M > 0$.

Moreover, in (1.1), we find that

$$
\frac{D}{Dt} = \frac{\partial}{\partial t} + V^* \cdot \nabla + \dot{\zeta}^* \frac{\partial}{\partial \zeta},
\tag{1.3}
$$