Doctopic: Partial Differential Equations ARTICLE IN PRESS YJMAA:21710

J. Math. Anal. Appl. ••• (••••) •••–•••

Contents lists available at ScienceDirect

Journal of Mathematical Analysis and Applications

www.elsevier.com/locate/jmaa

Spectral monodromy of small non-selfadjoint quantum perturbations of completely integrable Hamiltonians

Quang Sang Phan

Department of Mathematics, Faculty of Information Technology, Vietnam National University of Agriculture, Hanoi, Viet Nam

ARTICLE INFO

Article history:
Received 7 January 2017
Available online xxxx
Submitted by V. Radulescu

Keywords:
Integrable system
Non-selfadjoint
Spectral asymptotic
Pseudo-differential operators

ABSTRACT

We define a monodromy, directly from the spectrum of small non-selfadjoint perturbations of a selfadjoint semiclassical operator with two degrees of freedom, which is classically integrable. It is a combinatorial invariant that obstructs globally the existence of lattice structure of the spectrum, in the semiclassical limit. Moreover this spectral monodromy allows to recover a topological invariant (the classical monodromy) of the corresponding integrable system.

1. Introduction

1.1. Motivation

We propose in this article a way of detecting the monodromy of a quantum Hamiltonian which is classically integrable, by looking at non-selfadjoint perturbations.

In the classical theory, the classical monodromy is defined for a completely integrable system on symplectic manifolds as a topological invariant that obstructs the existence of global action-angle coordinates on the phase space, see Ref. [9].

Quantum monodromy was detected a long time ago in Ref. [7] and completely defined in Ref. [18], in the joint spectrum of system of selfadjoint operators that commute, in the sense of the semiclassical limit, as the classical monodromy of the underlying classical system.

However, a mysterious question is whether a monodromy can be defined for only one semiclassical operator? That is how to detect the modification of action-angle variables from only one spectrum?

We are interested in the globally structure of the spectrum of non-selfadjoint h-Weyl-pseudodifferential operators with two degrees of freedom, which are small non-selfadjoint perturbations of a selfadjoint operator, in the semiclassical limit. Such an operator is of the form

E-mail address: pqsang@vnua.edu.vn.

https://doi.org/10.1016/j.jmaa.2017.09.042
0022-247X/

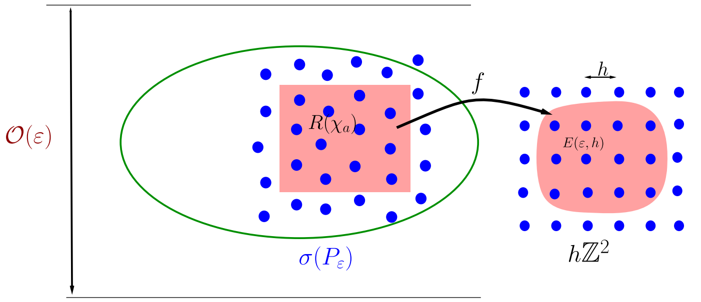

Fig. 1. h-local chart of the spectrum.

$$P_\varepsilon = P(x, hD_x, \varepsilon; h), \tag{1}$$

where the unperturbed operator $P := P_{\varepsilon=0}$ is formally selfadjoint, and ε is a small parameter. In this work ε is assumed to depend on the classical parameter h and in the regime $h \ll \varepsilon = \mathcal{O}(h^\delta)$, with $0 < \delta < 1$.

The first answer for the above problem, which was given in Ref. [13], is a particular case of the present work. In that work, the operators had the simple form $P + i\varepsilon Q$, such that the corresponding principal symbols p of P and q of Q commute for the Poisson bracket, $\{p, q\} = 0$. Here we develop this result in assuming only that the principal symbol of P_ε in (1) is of the form

$$p_\varepsilon = p + i\varepsilon q + \mathcal{O}(\varepsilon^2),$$

with p is a completely integrable Hamiltonian.

It is known from the spectral asymptotic theory (see Ref. [11]) that, under some suitable global assumption, the spectrum of the perturbed operators has locally the form of a deformed discrete lattice. The eigenvalues admit asymptotic expansions in h and ε. Moreover, we shall prove the main result (Theorem 4.4) that the spectrum is an *asymptotic pseudo-lattice* (see Definition 4.1). Therefore, as an application from Ref. [13], a combinatorial invariant of the spectral lattice – the *spectral monodromy* – is well defined, directly from the spectrum.

Moreover, this quantum result is strictly related to the classical results. The spectral monodromy can be identified to the classical monodromy of the completely integrable system p.

1.2. Brief description for the spectral monodromy

It is known from the spectral asymptotic theory (see Ref. [11]) that, under an ellipticity condition at infinity (see (13)), the spectrum of the perturbed operators (1) is discrete, and included in a horizontal band of size $\mathcal{O}(\varepsilon)$. Moreover, that work gives asymptotic expansions of the spectrum located in some small domains of size $\mathcal{O}(h^\delta) \times \mathcal{O}(\varepsilon h^\delta)$ of the spectral band, called *good rectangles*.

As we shall prove in this work that there is a correspondence (in fact a local diffeomorphism, see Proposition 4.6 and a proof for this point in Ref. [13]), denoted by f, from the spectrum contained in a good rectangle $R^{(a)}(\varepsilon, h)$ to a part of $h\mathbb{Z}^2$, modulo $\mathcal{O}(h^\infty)$,

$$R^{(a)}(\varepsilon, h) \ni \mu \mapsto f(\mu, \varepsilon; h) \in h\mathbb{Z}^2 + \mathcal{O}(h^\infty). \tag{2}$$

Here $a \in \mathbb{R}^2$ is introduced to fix the center of the good rectangle. This map is called a *h-local chart* of the spectrum. The spectrum therefore has the structure of a deformed lattice, with horizontal spacing h and vertical spacing εh. See Fig. 1.

Such good rectangles correspond to the Diophantine invariant tori in the phase space, on which the Hamiltonian flow of the unperturbed part, that is p, is quasi-periodic of constant frequency, see (29).

Please cite this article in press as: Q.S. Phan, Spectral monodromy of small non-selfadjoint quantum perturbations of completely integrable Hamiltonians, J. Math. Anal. Appl. (2018), https://doi.org/10.1016/j.jmaa.2017.09.042