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Original Article

Classes of pseudo BL-algebras with right Boolean lifting property

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Abstract

In this paper, we define the right Boolean lifting property (left Boolean lifting property) RBLP (LBLP) for pseudo BL-algebra to be the property that all Boolean elements can be lifted modulo every right filter (left filter) and next we study the behavior of RBLP (LBLP) with respect to direct products of pseudo BL-algebra. We introduce some conditions, which turn out to be a strengthening and a weakling of RBLP (LBLP) respectively and which open new ways of approaching the study of the RBLP (LBLP) in pseudo BL-algebras.

Keywords: Lifting property; Boolean center; (prime, maximal) filter; (maximal, local, hyper Archimedean, quasi-local, semi local) pseudo BL-algebra

1. Introduction

In 1998, Hajek introduced BL-algebra; an algebraic semantics of basic fuzzy logic which is generated by continuous t-norms on the interval [0, 1] and their residuals [1]. Then Georgescu introduced pseudo BL-algebra as a non-commutative extension of BL-algebra [2]. The idea of pseudo BL-algebra originates not only in logic and algebra, but also in algebraic properties that come from the syntax of certain non-classical propositional logics and intuitionistic logic. A lifting property for Boolean elements appears in the study of maximal MV-algebras and maximal BL-algebra. The left lifting property for Boolean elements modulo radical plays an essential part in the structure theorem for maximal pseudo BL-algebra. Extending previous works, Georgescu and Muresan studied Boolean lifting property for arbitrary residuated lattice [1]. The results of this study were similar to idempotent elements in the rings. In [3] we studied pseudo BL-algebra which satisfies the left (right) lifting property of Boolean elements modulo every left filter, a property that we have called LBLP (RBLP) for abbreviation. Also it shows that each Boolean algebra infused a pseudo BL-algebra with LBLP (RBLP), that hyper Archimedean have LBLP (RBLP).

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It turns out that the algebras at pseudo BL-algebra with LBLP (RBLP) are exactly the quasi-local pseudo BL-algebras. The target of this article is to study and introduce two conditions which share some properties with the RBLP (LBLP), (Propositions 5.12, 5.14 and 5.15) and appear to also differ by some properties from the RBLP (LBLP). Moreover, it shows that a finite direct product pseudo BL-algebra has RBLP iff each pseudo BL-algebra in the products has RBLP (LBLP) and this holds for individual filter, as well. Weaker results hold for arbitrary direct product of pseudo BL-algebra until mentioned otherwise, let $(A_i)_{i\in I}$ be a non-empty family of pseudo BL-algebras and $A = \prod_{i\in I} A_i$, since pseudo BL-algebra form an equational class, it follows that A becomes a pseudo BL-algebra with the operations defined canonically that is componentwise. Also, clearly, all elements are idempotent in A iff in A_i, for each $i \in I$. Throughout this section, unless mentioned otherwise, A will be an arbitrary pseudo BL-algebra. These are the main sources that inspire the research on pseudo BL-algebra. Section 2 shows theorems that satisfy the semi local condition and consists of previously known concepts about pseudo BL-algebra which are necessary in the next sections. In Section 3, we define the RBLP (LBLP) for pseudo BL-algebra and characterization of the RBLP (LBLP). In Section 4, we analyze the RBLP (LBLP) in direct products of pseudo BL-algebra. In Section 5, we set the RBLP (LBLP) in relation to two new arithmetic conditions. In Section 6, we establish relationships between the classes of the local, semi local, maximal, quasi-local, pseudo BL-algebras with RBLP (LBLP) and we obtain representation theorems for semilocal and maximal pseudo BL-algebras with RBLP (LBLP).

2. Preliminaries

In this section, we recall some basic definitions and results related to pseudo BL-algebra, all of them will be used in the paper. We shall denote by $\mathbb{N}$ the set of the natural numbers and by $\mathbb{N}^*$ the set of nonzero natural numbers.

Definition 2.1 (*[4]*). A pseudo BL-algebra is an algebra $(A, \vee, \wedge, \odot, \rightarrow, \rightsquigarrow, 0, 1)$ of type (2, 2, 2, 2, 2, 0, 0) satisfying the following

(PSBL_1) $(A, \vee, \wedge, 0, 1)$ is a bounded lattice;
(PSBL_2) $(A, \odot, 1)$ is a monoid;
(PSBL$_3$) $a \odot b \le c$ iff $a \le b \rightarrow c$ iff $b \le a \rightsquigarrow c$, for all $a, b, c \in A$;
(PSBL_4) $a \wedge b = (a \rightarrow b) \odot a = a \odot (a \rightsquigarrow b)$;
(PSBL_5) $(a \rightarrow b) \vee (b \rightarrow a) = (a \rightsquigarrow b) \vee (b \rightsquigarrow a) = 1$, for all $a, b \in A$.

Example 2.2. Let $a, b, c, d \in \mathbb{R}$, where $\mathbb{R}$ is the set of all real numbers. We put definition

$$(a, b) \le (c, d) \Longleftrightarrow a < c \text{ or } (a = c \text{ and } b \le d).$$

For any $a, b \in \mathbb{R} \times \mathbb{R}$, we define operations $\vee$ and $\wedge$ as follows: $a \vee b = max\{a, b\}$ and $a \wedge b = min\{a, b\}$. Let $A = \{(\frac{1}{2}, b) \in \mathbb{R}^2 : b \ge 0\} \cup \{(a, b) \in \mathbb{R}^2 : \frac{1}{2} < a < 1, b \in \mathbb{R}\} \cup \left\{(1, b) \in \mathbb{R}^2 : b \le 0\right\}$. For $(a, b), (c, d) \in A$, we put:

$$(a, b) \odot (c, d) = (\frac{1}{2}, 0) \vee (ac, bc + d),$$
$$(a, b) \rightarrow (c, d) = (\frac{1}{2}, 0) \vee [(\frac{c}{a}, \frac{d-b}{a}) \wedge (1, 0)],$$
$$(a, b) \rightsquigarrow (c, d) = (\frac{1}{2}, 0) \vee [(\frac{c}{a}, \frac{ad-bc}{a}) \wedge (1, 0)].$$

Then $(A, \vee, \wedge, \odot, \rightarrow, \rightsquigarrow, (\frac{1}{2}, 0), (1, 0))$ is a pseudo BL-algebra.

Proposition 2.3 (*[3]*). *If A is a pseudo BL-algebra and $a, b, c \in A$, then*

(psbl-c_1) $a \le b$ *iff* $a \rightarrow b = 1$ *iff* $a \rightsquigarrow b = 1$*;*
(psbl-c_2) $a \rightsquigarrow a = a \rightarrow a = 1$*;*
(psbl-c_3) $1 \rightsquigarrow a = 1 \rightarrow a = a$*;*
(psbl-c_4) $b \le a \rightsquigarrow b$ *and* $b \le a \rightarrow b$*;*