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Original article

On the solutions of quasi-static and steady vibrations equations in the theory of viscoelasticity for materials with double porosity

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Abstract

In the present paper the linear theory of viscoelasticity for Kelvin–Voigt materials with double porosity is considered. Some basic results on the solutions of the quasi-static and steady vibrations equations are obtained. Indeed, the fundamental solutions of the systems of equations of quasi-static and steady vibrations are constructed by elementary functions and their basic properties are established. Green's formulae and the integral representation of regular solution in the considered theory are obtained. Finally, a wide class of the internal boundary value problems of quasi-static and steady vibrations is formulated and on the basis of Green's formulae the uniqueness theorems for classical solutions of these problems are proved.

Keywords: Viscoelasticity; Double porosity; Fundamental solution; Uniqueness theorems; Quasi-static; Steady vibrations

1. Introduction

Poroelasticity is a well-developed theory for the interaction of fluid and solid phases of porous medium. The mathematical models of single- and multi-porosity media have found applications in many branches of civil and geotechnical engineering, biomechanics and technology (see e.g. Bai at al. [1], Cowin [2] and Vafai [3]).

The first theory of consolidation for elastic materials with single porosity is presented by Biot [4]. This theory is developed for double porosity elastic solid by Wilson and Aifantis [5]. More general models of double porosity materials are introduced by Ieşan and Quintanilla [6], Khalili et al. [7], Masters et al. [8], Gelet et al. [9] and studied by Ciarletta et al. [10], Straughan [11], Gentile and Straughan [12], Ieşan [13], Tsagareli and Svanadze [14]. An extensive review of works on the single- and multi-porosity elasticity and thermoelasticity is given in de Boer [15], Straughan [16–18].

Viscoelastic materials play an important role in many branches of engineering (see Brinson and Brinson [19], Gutierrez-Lemini [20], Lakes [21]). Various theories of differential and integral types of viscoelastic materials have

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been proposed by several authors (for details, see Amendola et al. [22], Christensen [23], Fabrizio and Morro [24], Eringen [25] and references therein).

In the last decade there has been interest in formulation of the theories of differential type for elastic materials with microstructures. In this connection, the theories of viscoelasticity and thermoviscoelasticity are presented for binary mixtures by Ieşan [26,27], Ieşan and Nappa [28], Ieşan and Quintanilla [29], Ieşan and Scalia [30]. The mathematical models for Kelvin–Voigt materials with single and double porosity are introduced in [31] and [32], respectively. The basic properties of plane waves are established and some boundary value problems of steady vibrations of the theories of viscoelasticity and thermoviscoelasticity for Kelvin–Voigt materials with double porosity are considered in [33,34].

In this paper the linear theory of viscoelasticity for Kelvin–Voigt materials with double porosity is considered. This work is articulated as follows. The next section is based on the governing field equations of quasi-statics and steady vibrations of the linear theory of viscoelasticity for Kelvin–Voigt materials with double porosity. In Section 3 the fundamental solutions of the systems of equations of quasi-static and steady vibrations are constructed by elementary functions and their basic properties are established. In Section 4 Green's formulae and the integral representation of regular solution in the considered theory are obtained. Finally, in Section 5 a wide class of the internal boundary value problems is formulated and on the basis of Green's formulae the uniqueness theorems for regular (classical) solutions of these problems are proved.

2. Basic equations

Let $\mathbf{x} = (x_1, x_2, x_3)$ be a point of the Euclidean three-dimensional space $\mathbb{R}^3$, let t denote the time variable, $t \geq 0$. We assume that subscripts preceded by a comma denote partial differentiation with respect to the corresponding Cartesian coordinate, repeated indices are summed over the range (1,2,3), and the dot denotes differentiation with respect to t.

In what follows we consider an isotropic and homogeneous viscoelastic Kelvin–Voigt material with double porosity that occupies the region Ω of $\mathbb{R}^3$. Let $\hat{\mathbf{u}}(\mathbf{x}, t)$ be the displacement vector, $\hat{\mathbf{u}} = (\hat{u}_1, \hat{u}_2, \hat{u}_3)$; $\hat{p}_1(\mathbf{x}, t)$ and $\hat{p}_2(\mathbf{x}, t)$ are the pore and fissure fluid pressures, respectively.

The system of dynamical equations in the linear theory of viscoelasticity for Kelvin–Voigt material with double porosity consists of the following equations [32]:

(a) *The equations of motion*

$$t_{lj,j} = \rho(\ddot{\hat{u}}_l - \hat{F}_l), \qquad l = 1, 2, 3, \tag{1}$$

where t_{lj} are the components of the total stress tensor, ρ is the reference mass density, $\rho > 0$, $\hat{\mathbf{F}} = (\hat{F}_1, \hat{F}_2, \hat{F}_3)$ is the body force per unit mass.

(b) *The equations of fluid mass conservation*

$$\begin{aligned} &\operatorname{div} \mathbf{v}^{(1)} + \dot{\zeta}_1 + \beta_1 \dot{e}_{rr} + \gamma(\hat{p}_1 - \hat{p}_2) = 0, \\ &\operatorname{div} \mathbf{v}^{(2)} + \dot{\zeta}_2 + \beta_2 \dot{e}_{rr} - \gamma(\hat{p}_1 - \hat{p}_2) = 0, \end{aligned} \tag{2}$$

where $\mathbf{v}^{(1)}$ and $\mathbf{v}^{(2)}$ are the fluid flux vectors for the pores and fissures, respectively; e_{lj} are the components of the strain tensor,

$$e_{lj} = \frac{1}{2}\left(\hat{u}_{l,j} + \hat{u}_{j,l}\right), \qquad l, j = 1, 2, 3, \tag{3}$$

β_1 and β_2 are the effective stress parameters, γ is the internal transport coefficient (leakage parameter) and corresponds to a fluid transfer rate respecting the intensity of flow between the pores and fissures, $\gamma \geq 0$; ζ_1 and ζ_2 are the increments of fluid (volumetric strain) in the pores and fissures, respectively, and defined by

$$\zeta_1 = \alpha_1 \hat{p}_1 + \alpha_3 \hat{p}_2, \qquad \zeta_1 = \alpha_3 \hat{p}_1 + \alpha_2 \hat{p}_2, \tag{4}$$

α_1 and α_2 measure the compressibilities of the pore and fissure systems, respectively; α_3 is the cross-coupling compressibility for fluid flow at the interface between the two pore systems at a microscopic level (see Khalili et al. [7], Masters et al. [8]).

(c) *The equations of effective stress concept*

$$t_{lj} = t'_{lj} - \left(\beta_1 \hat{p}_1 + \beta_2 \hat{p}_2\right) \delta_{lj}, \qquad l, j = 1, 2, 3, \tag{5}$$