Available online at www.sciencedirect.com
ScienceDirect

Transactions of A. Razmadze Mathematical Institute ▮ (▮▮▮▮) ▮▮▮–▮▮▮

Transactions of A. Razmadze Mathematical Institute

www.elsevier.com/locate/trmi

Original Article

Potential operators in modified Morrey spaces defined on Carleson curves

I.B. Dadashova[a], C. Aykol[b,*], Z. Cakir[b], A. Serbetci[b]

[a] Baku State University, Baku, Azerbaijan
[b] Ankara University, Department of Mathematics, Ankara, Turkey

Received 4 August 2017; received in revised form 26 September 2017; accepted 27 September 2017
Available online xxxx

Abstract

In this paper we study the potential operator I^{α}_{Γ}, $0 < 1$ in the modified Morrey space $\widetilde{L}_{p,\lambda}(\Gamma)$ and the spaces $BMO(\Gamma)$ defined on Carleson curves Γ. We prove that for $1 < p < (1-\lambda)/\alpha$ the potential operator I^{α}_{Γ} is bounded from the modified Morrey space $\widetilde{L}_{p,\lambda}(\Gamma)$ to $\widetilde{L}_{q,\lambda}(\Gamma)$ if and in the case of infinite curve only if $\alpha \le 1/p - 1/q \le \alpha/(1-\lambda)$, and from the spaces $\widetilde{L}_{1,\lambda}(\Gamma)$ to $W\widetilde{L}_{q,\lambda}(\Gamma)$ if and in the case of infinite curve only if $\alpha \le 1 - \frac{1}{q} \le \frac{\alpha}{1-\lambda}$. Furthermore, for the limiting case $(1-\lambda)/\alpha \le p \le 1/\alpha$ we show that if Γ is an infinite Carleson curve, then the modified potential operator $\widetilde{\mathcal{I}}^{\alpha}_{\Gamma}$ is bounded from $\widetilde{L}_{p,\lambda}(\Gamma)$ to $BMO(\Gamma)$, and if Γ is a finite Carleson curve, then the operator I^{α}_{Γ} is bounded from $\widetilde{L}_{p,\lambda}(\Gamma)$ to $BMO(\Gamma)$.

Keywords: Carleson curves; Modified Morrey spaces; BMO spaces; Potential operators; Sobolev–Morrey inequality

1. Introduction

Morrey spaces were introduced by C.B. Morrey [1] in 1938 in connection with certain problems in elliptic partial differential equations and calculus of variations (see [2,3]). Later, Morrey spaces found important applications to Navier–Stokes and Schrödinger equations, elliptic problems with discontinuous coefficients and potential theory (see [4–8]).

Let $\Gamma = \{t \in \mathbb{C} : t = t(s),\ 0 \le s \le l \le \infty\}$ be a rectifiable Jordan curve in the complex plane with arc-length measure $\nu(t) = s$, here $l = \nu\Gamma =$ lengths of Γ.

* Corresponding author.
E-mail addresses: irada-dadashova@rambler.ru (I.B. Dadashova), Canay.Aykol@science.ankara.edu.tr (C. Aykol), zeyncakir@gmail.com (Z. Cakir), serbetci@ankara.edu.tr (A. Serbetci).
Peer review under responsibility of Journal Transactions of A. Razmadze Mathematical Institute.

https://doi.org/10.1016/j.trmi.2017.09.004

We denote

$$\Gamma(t,r)=\Gamma\cap B(t,r),\ t\in\Gamma,\ r>0,$$

where $B(t,r)=\{z\in\mathbb{C}:|z-t|<r\}$.

A rectifiable Jordan curve Γ is called a Carleson curve (regular curve) if the condition

$$\nu\Gamma(t,r)\le c_0 r$$

holds for all $t\in\Gamma$ and $r>0$, where the constant $c_0>0$ does not depend on t and r.

Definition 1. Let $1\le p<\infty, 0\le\lambda\le 1, [r]_1=\min\{1,r\}$. We denote by $L_{p,\lambda}(\Gamma)$ the Morrey space, and by $\widetilde{L}_{p,\lambda}(\Gamma)$ the modified Morrey space, the set of locally integrable functions f on Γ with the finite norms

$$\|f\|_{L_{p,\lambda}(\Gamma)}=\sup_{t\in\Gamma}\sup_{r>0}r^{-\frac{\lambda}{p}}\|f\|_{L_p(\Gamma(t,r))},$$

$$\|f\|_{\widetilde{L}_{p,\lambda}(\Gamma)}=\sup_{t\in\Gamma}\sup_{r>0}[r]_1^{-\frac{\lambda}{p}}\|f\|_{L_p(\Gamma(t,r))}$$

respectively.

Note that

$$L_{p,0}(\Gamma)=\widetilde{L}_{p,0}(\Gamma)=L_p(\Gamma),$$

$$\widetilde{L}_{p,\lambda}(\Gamma)\subset_{\succ}L_{p,\lambda}(\Gamma)\cap L_p(\Gamma)\quad\text{and}\quad\max\{\|f\|_{L_{p,\lambda}(\Gamma)},\|f\|_{L_p(\Gamma)}\}\le\|f\|_{\widetilde{L}_{p,\lambda}(\Gamma)}\tag{1}$$

and if $\lambda<0$ or $\lambda>1$, then $L_{p,\lambda}(\Gamma)=\widetilde{L}_{p,\lambda}(\Gamma)=\Theta$, where Θ is the set of all functions equivalent to 0 on Γ.

We denote by $WL_{p,\lambda}(\Gamma)$ the weak Morrey space, and by $W\widetilde{L}_{p,\lambda}(\Gamma)$ the modified Morrey space, as the set of locally integrable functions f on Γ with finite norms

$$\|f\|_{WL_{p,\lambda}(\Gamma)}=\sup_{\beta>0}\beta\sup_{r>0,\,t\in\Gamma}\left(r^{-\lambda}\int_{\{\tau\in\Gamma(t,r):\,|f(\tau)|>\beta\}}d\nu(\tau)\right)^{1/p},$$

$$\|f\|_{W\widetilde{L}_{p,\lambda}(\Gamma)}=\sup_{\beta>0}\beta\sup_{r>0,\,t\in\Gamma}\left([r]_1^{-\lambda}\int_{\{\tau\in\Gamma(t,r):\,|f(\tau)|>\beta\}}d\nu(\tau)\right)^{1/p}.$$

Note that

$$WL_p(\Gamma)=WL_{p,0}(\Gamma),\ L_{p,\lambda}(\Gamma)\subset WL_{p,\lambda}(\Gamma)\text{ and }\|f\|_{WL_{p,\lambda}(\Gamma)}\le\|f\|_{L_{p,\lambda}(\Gamma)}.$$

Definition 2. The space of functions with bounded mean oscillation $BMO(\Gamma)$ is defined as the set of locally integrable functions f with finite norm

$$\|f\|_{BMO(\Gamma)}=\sup_{r>0,\ t\in\Gamma}(\nu\Gamma(t,r))^{-1}\int_{\Gamma(t,r)}|f(\tau)-f_{\Gamma(t,r)}|d\nu(\tau)<\infty,$$

where

$$f_{\Gamma(t,r)}=(\nu\Gamma(t,r))^{-1}\int_{\Gamma(t,r)}f(\tau)d\nu(\tau).$$

Maximal operators and potential operators in various spaces defined on Carleson curves have been widely studied by many authors (see, for example [9–16]). In Morrey spaces defined on quasimetric measure spaces, in particular Morrey spaces $L_{p,\lambda}(\Gamma)$ defined on Carleson curves N. Samko [16] studied the boundedness of the maximal operator M_Γ defined by

$$M_\Gamma f(t)=\sup_{t>0}(\nu\Gamma(t,r))^{-1}\int_{\Gamma(t,r)}|f(\tau)|d\nu(\tau)$$

and proved the following:

Please cite this article in press as: I.B. Dadashova, et al., Potential operators in modified Morrey spaces defined on Carleson curves, Transactions of A. Razmadze Mathematical Institute (2017), https://doi.org/10.1016/j.trmi.2017.09.004.