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Original article

The stability of orthotropic shells of revolution, close to cylindrical ones, with an elastic filler, under the action of torsion, normal pressure and temperature

Sergo Kukudzhanov

A. Razmadze Mathematical Institute of I. Javakhishvili Tbilisi State University, 6 Tamarashvili Str., Tbilisi 0177, Georgia

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Abstract

The paper investigates the stability of orthotropic shells of revolution which are by their form close to cylindrical ones, with an elastic filler, under the action of torques, external pressure and temperature. The shell is assumed to be thin and elastic. Temperature is uniformly distributed in the shell body. The filler is simulated by an elastic base. The shells of positive and negative Gaussian curvature are considered. Formulas for finding critical loadings and corresponding forms of stability loss are derived.

Keywords: Stability; Shells; Filler; Critical load; Curvature; Temperature

In the present paper we investigate the problem of the stability of closed shells of revolution which are by their form close to cylindrical ones, with an elastic filler, under the action of torques moments M applied to the shell ends, external pressure q distributed over the lateral surface, and temperature. We consider a light filler for which the influence of tangential stresses on the contact surface may be neglected. Temperature is uniformly distributed in the shell body. An elastic filler is simulated by Winkler's base, its extension under the heating is not taken into account. We consider the shells of middle length whose midsurface generatrix is described by a parabolic function, and also the shells of positive and negative Gaussian curvature. The boundary conditions on the end-walls correspond to a free support admitting certain radial displacement in the initial state. In addition, like cylindrical shells, when investigating the forms of stability loss special attention has been paid only to the principal boundary conditions whose fulfilment allowed one to get good enough value approximation of critical loading for freely supported end-walls [1,2].

In a dimensionless form we present formulas and universal curves whose critical values of shearing and normal forces depend on the orthotropy parameters, elastic filler rigidity, temperature and dimensionless amplitude of

E-mail address: kotic13@mail.ru.
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deviation from the cylindrical form. It is shown that the elastic orthotropy parameters significantly affect critical torsion and normal pressure loading. It should also be mentioned that they differently affect critical loading and form of wave formation depending on the sign of Gaussian curvature and deviation amplitude. For convex shells, as the deviation amplitude increases, the influence of an elastic constant increases in axial direction, whereas for concave shells, the influence of an elastic constant increases in circumferential direction, but only for small values of filler rigidity. For comparatively not higher temperature, as the filler rigidity increases, the critical loading increases. However, under higher temperature, for a certain value of the filler rigidity and further its growth, the critical loading decreases, and this is an important factor for practice.

We consider the shell whose midsurface is formed by rotating the square parabola around the z-axis of the rectangular system of coordinates x, y, z with the origin at the midsegment of the axis of revolution. It is assumed that radius r of the midsurface cross-section of the shell is defined by the equality $R = r + \delta_0\left[1 - \zeta^2(r/l)^2\right]$, where r is radius of the end-wall cross-section, δ_0 is the maximal deviation (for $\delta_0 > 0$, the shell is convex, and for $\delta_0 < 0$ it is concave), $L = 2l$ is the shell length, $\xi = z/r$ $(-l \le z \le l)$. It is assumed that

$$(\delta_0/r)^2 \cdot (\delta_0/l)^2 \ll 1. \tag{1}$$

Temperature is assumed to be uniformly distributed in the shell body. An elastic filler is simulated by Winkler's base, its extension upon heating is not taken into account.

As the basic equations of thermostability we adopted those of the theory of shallow orthotropic shells [3]. For the midlength shells under consideration [4], the stability loss is accompanied by a weakly expressed wave formation in a longitudinal direction compared with the circumferential one, therefore the relation

$$\frac{\partial^2 f}{\partial \xi^2} \ll \frac{\partial^2 f}{\partial \varphi^2} \quad (f = w, \psi) \tag{2}$$

is valid, where w and ψ are the functions of radial displacement and tension, respectively. As a result, the system of equations for the shells under consideration is reduced to the following equation [5] (owing to the adopted assumption, temperature terms are equal to zero [6]):

$$\varepsilon \frac{\partial^8 w}{\partial \varphi^8} + \frac{E_1}{E_2}\left(\frac{\partial^4 w}{\partial \xi^4} + 4\delta \frac{\partial^4 w}{\partial \varphi^4} + 4\delta^2 \frac{\partial^4 w}{\partial \varphi^4}\right) - \left(\frac{T_1^0}{E_2 h}\frac{\partial^6 w}{\partial \xi^2 \partial \varphi^4} + \frac{T_2^0}{E_2 h}\frac{\partial^6 w}{\partial \varphi^6} + \frac{2S^0}{E_2 h}\frac{\partial^6 w}{\partial \xi \partial \varphi^5}\right) + \frac{\beta r^2 \partial^4 w}{E_2 h \partial \varphi^4} = 0 \tag{3}$$

$$\varepsilon = h^2/12r^2(1 - \nu_1 \nu_2), \qquad \delta = \delta_0 r/l^2.$$

Here, E_1, E_2, ν_1 ν_2 are, respectively, elasticity modules and Poisson coefficients in the axial and circumferential directions ($E_1\nu_2 = E_2\nu_1$); T_1^0 and T_2^0 are, respectively, axial and circumferential normal stresses of subcritical state; S^0 is shearing stress of subcritical state; β is the "bed" coefficient of the elastic filler in supercritical state; h is the shell thickness; φ is the angular coordinate. The subcritical state is assumed to be momentless. Relying on the corresponding solution and taking into account the filler reaction and inequalities (1), we obtain the following approximate expressions:

$$T_1^0 = q\delta_0\left[\xi^2(r/l)^2 - 1\right], \quad T_2^0 = -qr + \beta_0 r w_0, \quad S^0 = M/2\pi r$$

where w_0, β_0 are, respectively, deflection and the "bed" coefficient of the filler in the initial state; M is torque; q is external pressure ($q > 0$). In addition,

$$\frac{T_1^0}{\varepsilon_2 h} = \frac{\sigma_1^0}{E_2} = \frac{q\delta_0}{E_2 h}(\xi^2(r/l)^2 - 1), \qquad \frac{T_2^0}{E_2 h} = \frac{\sigma_2^0}{E_2} = -\frac{qr}{E_2 h} + w_0\frac{\beta_0 r}{E_2 h}. \tag{4}$$

Taking into account that in the initial position the shell deformation in circumferential direction is defined by the equalities

$$\varepsilon_\varphi = \frac{\sigma_2^0 - \nu\sigma_1^0}{E_2} + \alpha_2 T, \qquad \varepsilon_\varphi = -\frac{w_0}{r},$$

where α_2 is the coefficient of linear extension in circumferential direction and T is temperature, we obtain

$$w_0 = (-\sigma_2^0 + \nu_1\sigma_1^0)\frac{r}{E_2} - \alpha_2 r T. \tag{5}$$

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