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Original article

Generalized semi-open and pre-semiopen sets via ideals

Bishwambhar Roy[a,*], Ritu Sen[b]

[a] *Department of Mathematics, Women's Christian College, 6, G. C. Row, Kolkata — 700 026, India*
[b] *Department of Mathematics, Presidency University, Kolkata — 700073, India*

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Abstract

In this paper we have introduced a new type of sets termed as μ^*-open sets which unifies semiopen sets, β-open sets and discussed some of its properties. We have also introduced another type of weak open sets termed as $\mathcal{I}_\mu$-open sets depending on a GT as well as an ideal on a topological space. Finally the concept of weakly $\mathcal{I}_\mu$-open sets are investigated.

Keywords: μ-open set; Ideal; μ^*-open set; $\mathcal{I}_\mu$-open set; Weakly $\mathcal{I}_\mu$-open set

1. Introduction

The concept of ideal on topological spaces was studied by Kuratowski [1] and Vaidyanathaswamy [2] which is one of the important areas of research in the branch of mathematics. After them different mathematicians applied the concept of ideals in topological spaces (see [2–8]). In the past few years mathematicians turned their attention towards the generalized open sets (see [8–12] for details). Our aim in this paper is to use the concept of ideals in the generalized topology introduced by A. Császár. We recall some notions defined in [10].

Let $expX$ denote the power set of a non-empty set X. A class μ ($\subsetneqq expX$) is called a generalized topology [10], (briefly, GT) if $\varnothing \in \mu$ and μ is closed under arbitrary union. The elements of μ are called μ-open sets and the complement of μ-open sets are known as μ-closed sets. A set X with a GT μ on it is known as a generalized topological space (briefly, GTS) and is denoted by (X, μ). A GT μ is said to be a quasi topology (briefly QT) [17] if $M, M' \in \mu$ implies $M \cap M' \in \mu$. The pair (X, μ) is said to be a QTS if μ is a QT on X.

For any $A \subsetneqq X$, the generalized μ-closure of A is denoted by $c_\mu(A)$ and is defined by $c_\mu(A) = \cap\{F : F$ is μ-closed and $A \subsetneqq F\}$, similarly $i_\mu(A) = \cup\{U : U \subsetneqq A$ and $U \in \mu\}$ (see [10,11]). Throughout the paper μ, λ will always mean GT on the respective sets.

* Corresponding author.
E-mail addresses: bishwambhar_roy@yahoo.co.in (B. Roy), ritu_sen29@yahoo.co.in (R. Sen).
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An ideal [1] $\mathcal{I}$ on a topological space (X, τ) is a non-empty collection of subsets of X with the following properties: (i) $A \subsetneqq B$ and $B \in \mathcal{I} \Rightarrow A \in \mathcal{I}$ (ii) $A \in \mathcal{I}, B \in \mathcal{I} \Rightarrow A \cup B \in \mathcal{I}$. An ideal $\mathcal{I}$ on a topological space (X, τ) is denoted by $(X, \tau, \mathcal{I})$ and known as an ideal topological space.

2. μ^*-open sets

Definition 2.1. Let μ be a GT on a topological space (X, τ). A subset A of X is called μ^*-open [13] if $A \subsetneqq cl(i_\mu(A))$.

Theorem 2.2. *Let μ be a GT on a topological space (X, τ). Then A is μ^*-open if and only if there exists a μ-open set U such that $U \subsetneqq A \subsetneqq cl(U)$.*

Proof. Let A be a μ^*-open set. Then $A \subsetneqq cl(i_\mu(A))$. Let $U = i_\mu(A)$. Then U is μ-open and $U \subsetneqq A \subsetneqq cl(i_\mu(A)) = cl(U)$. Conversely, let there exist a μ-open set U such that $U \subsetneqq A \subsetneqq cl(U)$. Then $U \subsetneqq A \Rightarrow U \subsetneqq i_\mu(A) \Rightarrow cl(U) \subsetneqq cl(i_\mu(A)) \Rightarrow A \subsetneqq cl(i_\mu(A))$. Thus A is μ^*-open.

Remark 2.3. Let μ be a GT on a topological space (X, τ). If

(i) $\mu = \tau$, then μ^*-open set reduces to semiopen set [14];

(ii) $\mu = PO(X)$, then μ^*-open set reduces to β-open set [15];

(iii) every μ-open set is μ^*-open;

(iv) If λ be any other GT on X with $\mu \subsetneqq \lambda$, then every μ^*-open set is λ^*-open.

Note 2.4. Let μ be a GT on a topological space (X, τ). Then the collection of all μ^*-open sets forms a GT on X.

Proof. Clearly $\varnothing$ is a μ^*-open set. Let $\{A_\alpha : \alpha \in \Lambda\}$ be a family of μ^*-open sets. Then there exist μ-open sets U_α such that $U_\alpha \subsetneqq A_\alpha \subsetneqq cl(U_\alpha)$ for each $\alpha \in \Lambda$. Thus $\cup\{U_\alpha : \alpha \in \Lambda\} = U$ (say) $\subsetneqq \cup\{A_\alpha : \alpha \in \Lambda\} \subsetneqq cl(U)$ where U is μ-open showing that the union of μ^*-open sets is μ^*-open.

Example 2.5. (a) Let $X = \{a, b, c\}$, $\tau = \{\varnothing, \{a\}, \{a, c\}, X\}$ and $\mu = \{\varnothing, \{c\}, \{a, c\}\}$. Then μ is a GT on the topological space (X, τ). It can be checked easily that $\{b, c\}$ is a μ^*-open set which is not a μ-open set.

(b) Let $X = \{a, b, c\}$, $\mu = \{\varnothing, \{a, b\}, \{a, c\}, \{b, c\}, X\}$ and $\tau = \{\varnothing, \{a\}, \{b\}, \{a, b\}, X\}$. Then μ is a GT on the topological space (X, τ). It can be easily verified that $\{a, b\}$ and $\{a, c\}$ are both μ^*-open but their intersection $\{a\}$ is not so.

Theorem 2.6. *Let μ be a GT on a topological space (X, τ) and A be a μ^*-open set such that $A \subsetneqq B \subsetneqq cl(A)$. Then B is also a μ^*-open set.*

Proof. As A is μ^*-open, there exists a μ-open set U such that $U \subsetneqq A \subsetneqq cl(U)$. Thus $U \subsetneqq B$. Also $cl(A) \subsetneqq cl(U) \Rightarrow B \subsetneqq cl(U)$. Thus $U \subsetneqq B \subsetneqq cl(U)$. Thus B is μ^*-open.

3. $\mathcal{I}_\mu$-open sets

Definition 3.1. Let μ be a GT on an ideal topological space $(X, \tau, \mathcal{I})$. A subset A of X is called $\mathcal{I}_\mu$-open if there exists a μ-open set U such that $U \setminus A \in \mathcal{I}$ and $A \setminus cl(U) \in \mathcal{I}$.

If $A \in \mathcal{I}$, then A is an $\mathcal{I}_\mu$-open set and also by Theorem 2.2, every μ^*-open set (hence every μ-open set) is $\mathcal{I}_\mu$-open for any ideal $\mathcal{I}$ on X.

Example 3.2. (a) Let $X = \{a, b, c\}$, $\mu = \{\varnothing, \{a\}, \{a, b\}, \{a, c\}, \{b, c\}, X\}$, $\tau = \{\varnothing, \{a\}, \{a, b\}, \{a, c\}, X\}$ and $\mathcal{I} = \{\varnothing, \{a\}, \{b\}, \{a, b\}\}$. Then μ is a GT on the ideal topological space $(X, \tau, \mathcal{I})$. It can be verified that $\{b\}$ is $\mathcal{I}_\mu$-open but not μ^*-open.