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Original Article

Conservation of time scale for one-dimensional pulsating flow

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Abstract

In the work by analysis of one-dimensional unsteady flows, based on the fundamental law of conservation with application of Fourier series is shown that in the presence of periodic, steady pulsations along the flow, the main frequency as well as all higher frequencies remain constant and only the amplitude of oscillations is changed that is in full agreement with the results of analysis of more complex three-dimensional flows. Thus, is confirmed the validity of the principle of conservation of frequencies or time scale along the flow. So, is obtained very interesting result for turbulence problem solution.

Keywords: Oscillation; Turbulence; Time scale; Conservation

1. Introduction

Integrating the Navier–Stokes differential equation, Osborne Reynolds admitted that:

$$\overline{\nabla F} = \nabla\left(\overline{F}\right). \tag{1.1}$$

In the work [1] was shown, that one of the main reasons of the Reynolds problem arising is this assumption. If we have an arbitrary periodic function:

$$\overline{F} = \frac{1}{\tau_0}\int_0^{\tau_0} F\left(x, y, z, t\right) dt. \tag{1.2}$$

The following relations are valid:

$$\overline{\nabla F} = \nabla\left(\overline{F}\right) + \frac{\overline{F}}{\tau_0}\nabla\tau_0 = \nabla\left(\overline{F}\right) - \overline{F}A. \tag{1.3}$$

$$\overline{\nabla^2 F} = \nabla^2\left(\overline{F}\right) - 2A\left[\nabla\left(\overline{F}\right)\right] - \overline{F}\left(\nabla A\right) + \overline{F}A^2, \tag{1.4}$$

where-τ_O is the duration of oscillation. $A = (1/f)\, grad\,(f) = grad\,(\ln f) = -grad\,(\ln\tau_0)$.

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Integrating the differential equations of Navier–Stokes by taking in account (1.2)–(1.4), are obtained differential equations, that differ from the Reynolds equation. At the same time, the Reynolds equations express conservation laws for integral flows and they do not cause doubt. Consequently, the presence of two different systems makes it possible to obtain very important additional information on the turbulence problem. One of these results is the principle of conservation of frequencies (or time scales) along the flow.

In this paper, we prove what has been said on the example of a one-dimensional nonstationary periodic flow.

From acoustic theory it is well known that at propagation of acoustic waves, pressure fluctuation period and character at various locations of the perturbation region are qualitatively identical [2,3]. With increasing of distance from the source of vibration the amplitude of perturbations changes due to dissipation at perturbations spreading in a large space (in the case of spherical waves), however, the period of oscillation at this is not changing. Therefore, audio signals are not distorted, despite that they become weaker. In terms of acoustics theory, mathematically this would be easily explained, since perturbations that are propagating with the constant speed C should create the same pattern in different locations of space with shift in time x/C (see solutions of wave equations). Thus, we can say that for the case of acoustic disturbances, the preservation of oscillation frequency is observed. However, let us put the question of whether or not to preserve as constant the oscillation frequency along the flow, if we have arbitrary, strong periodic disturbances? The theory of acoustic waves in this issue does not help us, because, at significant perturbations, the wave propagation velocity is not constant due to its dependence on the changing of the environment state parameters.

However, as will be shown below, if in the one-dimensional flow are propagated periodic waves of arbitrary shape, the frequency of these oscillations in arbitrary cross section also will be the same. In other words, we show that conservation of frequency along certain lines is a property not only of acoustic disturbances, but also of arbitrary non-stationary periodic processes. Starting from simple examples, with the transition to a more general problem, we show that this property is a common feature of all periodic oscillatory processes. Therefore, this feature would be called as principle of conservation of frequencies (or time scales) along the vector of substance that is subject of periodic fluctuations.

2. Basic part

For obviousness, let us assume that in the straight channel receives periodic stream. If in the initial section of the channel we install the pressure sensor, it will register the oscillation process with the period of τ_0 (Fig. 1, line 1) or with the frequency $f = 1/\tau_0 = \omega/2\pi$. For these processes, there is a conventional, minimum angular velocity that will be determined from the equation $\omega = 2\pi f = 2\pi/\tau_0$.

The sensor located in a certain distance from the entrance section will also detect a certain periodic process with interval τ_x, and the perturbation amplitude will be relatively less (line 2). But third sensor that is located very far from the entrance, almost will not register vibrations due to dissipation and smoothing of the waves, the flow will gradually make stationary character (line 3).

We will show that, in arbitrary section of one-dimensional periodic flow, the oscillation period of the pulsating flow parameters must be the same not only for small but also for any perturbations ($\tau_0 = \tau_x = idem$). In other words, in any section, the period of the vibration and main angular velocity will be the same

$$\frac{\partial \omega}{\partial x} = 0, \tag{2.1}$$

$$\frac{\partial \tau_x}{\partial x} = 0 \tag{2.2}$$

to confirm the above mentioned, let us consider the instantaneous value of mass flow in an arbitrary cross-section of flow $G = \rho U F$. The instantaneous specific mass flow would be written as the sum of two functions, one of that depends on the coordinate x and the other is a periodic function, (and dependent on x and t)

$$g = \rho U = \eta(x) + \varphi(x, t), \tag{2.3}$$

thus, a periodic function is possible to express as a Fourier series [4,5]

$$\varphi(x, \tau) = \sum_{i=1,\infty} \left[a(x)_i \cos(i\omega t) + b(x)_i \sin(i\omega t)\right], \tag{2.4}$$

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