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Original article

Method of fundamental solutions for mixed and crack type problems in the classical theory of elasticity

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Abstract

We analyse some new aspects concerning application of the fundamental solution method to the basic three-dimensional boundary value problems, mixed transmission problems, and also interior and interfacial crack type problems for steady state oscillation equations of the elasticity theory. First we present existence and uniqueness theorems of weak solutions and derive the corresponding norm estimates in appropriate function spaces. Afterwards, by means of the columns of Kupradze's fundamental solution matrix special systems of vector functions are constructed explicitly. The linear independence and completeness of these systems are proved in appropriate Sobolev–Slobodetskii and Besov function spaces. It is shown that the problem of construction of approximate solutions to the basic and mixed boundary value problems and to the interior and interfacial crack problems can be reduced to the problems of approximation of the given boundary vector functions by elements of the linear spans of the corresponding complete systems constructed by the fundamental solution vectors. By this approach the approximate solutions of the boundary value and transmission problems are represented in the form of linear combinations of the columns of the fundamental solution matrix with appropriately chosen poles distributed outside the domain under consideration. The unknown coefficients of the linear combinations are defined by the approximation conditions of the corresponding boundary and transmission data.

Keywords: Method of fundamental solutions; Theory of elasticity; Elastic vibrations; Mixed boundary value problem; Mixed transmission problem; Crack problem; Approximate solutions

1. Introduction

The Method of Fundamental Solutions (MFS) for partial differential equations was first proposed by V. Kupradze in the 1960s (see the pioneering works in this direction by V. Kupradze and M. Alexidze, [1,2], [KuAl]). The main

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idea of the MFS is to distribute the singularity poles $\{y^{(k)}\}_{k=1}^{\infty}$ of the fundamental solution $\Gamma(x-y)$ of a differential operator outside the domain under consideration, construct the set of functions $\{\Gamma(x-y^{(k)})\}_{k=1}^{\infty}$, prove its density properties in appropriate function spaces, and then approximate the sought-for solution by a linear combination of the fundamental solutions, $\sum_{k=1}^{N} C_k\, \Gamma(x-y^{(k)})$ with unknown coefficients C_k, which are to be determined by satisfying the corresponding boundary conditions.

Starting from the 1970s, the MFS gradually became a useful technique and is used to solve a large variety boundary value problems (BVP) arising in the mathematical models of physics, engineering, and biomedicine (see [1–18], and the references therein). However, it should be mentioned that until now it has not been worked out how to apply the MFS to crack type problems in solid mechanics, since the different approaches related to MFS described in the scientific literature are not applicable to crack type problems. To work out this problematic topic and to extend the MFS to crack type boundary-value problems are among the main goals of the present investigation. We will reformulate crack type problems in the form of mixed type transmission problems introducing an artificial interface boundary containing the crack faces and then substantiate mathematically the MFS on the basis of the results obtained for mixed transmission problems.

For the basic and mixed exterior boundary value problems, as well as for the crack and mixed transmission problems of steady state elastic oscillations, here we develop the approach which is applicable for all values of the oscillation frequency parameter.

We have to mention here that the main shortage of the MFS is its poor conditioning which should be alleviated, e.g., by preconditioning of the corresponding system matrix or by iterative refinement or by some other artificial approaches available for special particular cases (see, e.g. [19]).

However, the MFS features remarkable and unusual ease of implementation due to the following reasons (see, e.g. [3,17,19]): *"Uniform character of the trial functions, complete absence of singular integral evaluations, it does not require an elaborate discretization of the boundary, simplicity of finding values of approximate solution at inner points of the domain of interest, the derivatives of the MFS approximation can also be evaluated directly, extreme abundance of the set of trial functions that results in a high adaptivity of the method, MFS can be applied even in the case of domains with irregular boundaries (e.g., for domains with Lipschitz boundaries)"*. More detailed overview of the results related to the fundamental solution method can be found in [17] and the references therein.

In this paper we prove linear independence and density property of the appropriately chosen systems of vector functions constructed by the corresponding fundamental solutions (Kupradze's matrix of fundamental solutions). These systems are associated with particular type of problems and actually they reduce the solving procedure of boundary value problems to the approximation problems of the boundary data in the appropriate non-orthogonal complete systems of vector functions.

The paper is organized as follows. In Section 2, we introduce the notions of regular, semi-regular and weak solutions and formulate classical and weak settings of boundary value and transmission problems for steady state oscillation equations of the elasticity theory. We formulate also the corresponding uniqueness theorems for the problems under consideration in the class of vector functions satisfying the Sommerfeld–Kupradze radiation conditions at infinity. In Section 3, existence and uniqueness theorems are proved for weak solutions and the corresponding estimates are obtained in appropriate function spaces. Section 4 is devoted to the fundamental solution method for basic and mixed boundary value problems, as well as for the basic and mixed transmission problems containing crack type problems as special particular cases. Special systems of vector functions are constructed explicitly by means of the columns of Kupradze's fundamental solution matrix and their linear independence and completeness are proved in appropriate Sobolev–Slobodetskii and Besov function spaces. The problem of construction of approximate solutions to the boundary value and transmission problems are reduced to the approximation problems of the given boundary vector functions by linear combinations of the elements of the corresponding nonorthogonal, linearly independent, complete vector systems. In Appendix A, we collect some auxiliary material needed in the main text of the paper concerning properties of layer potentials and the corresponding boundary operators. In Appendix B, we present alternative integral representations of radiating solutions in unbounded regions. Finally, in Appendix C, we recall some results from the theory of strongly elliptic pseudodifferential equations on manifolds with boundary in Bessel potential and Besov spaces which are the main tools for proving existence theorems for mixed boundary, boundary-transmission, and crack type problems by the potential methods.

The approach developed in this paper can be successfully applied to boundary value problems of mathematical physics for homogeneous and piecewise homogeneous bounded and unbounded composite media containing interior

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