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Original article

# Mathematical problems of thermoelasticity of bodies with microstructure and microtemperatures

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**Abstract**

The paper deals with the linear theory of thermoelasticity for elastic isotropic microstretch materials with microtemperatures and microdilatations. For the differential equations of pseudo-oscillations the fundamental matrix is constructed explicitly in terms of elementary functions. With the help of the corresponding Green identities the general integral representation formula of solutions by means of generalized layer and Newtonian potentials are derived. The basic Dirichlet and Neumann type boundary value problems are formulated in appropriate function spaces and the uniqueness theorems are proved. The existence theorems for classical solutions are established by using the potential method.


*Keywords:* Elastic bodies with microstructure; Thermoelasticity with microtemperatures; Potential theory; Integral equations

## 1. Introduction

The main goal of our investigation is analysis of the basic boundary value problems for the pseudo-oscillation equations of the theory of thermoelasticity for isotropic materials with microstructure, whose microelements possess microtemperatures.

A theory of thermoelasticity with microtemperatures, in which the microelements can stretch and contract independently of their translations has been studied by Ieşan [1]. This is the simplest thermomechanical theory of elastic bodies that takes into account the microtemperatures and the inner structure of the materials. This model has been investigate by various authors (see e.g., [2–4]).

The mathematical model of a linear theory of thermodynamics for microstretch elastic solids with microtemperatures, using the results established by Grot [5] has been proposed by Ieşan [6]. This theory introduces three extra degrees of freedom over the theory presented in [1]. An interesting aspect in this theory is the coupling of

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microrotation vector with the microtemperatures even for isotropic bodies. This effect is different from the classical theory of Cosserat thermoelasticity for isotropic bodies [7], where the microrotation vector is independent of the thermal field. In the model [6] a material particle is equipped with 11 degrees of freedom (3 displacement components, 3 microrotation components, 3 microtemperature components, 1 microdilatation and 1 temperature).

The system of differential equations of thermodynamics for isotropic elastic materials with microstructure, with respect to the displacement vector, microrotation vector, microtemperature vector, microdilation function, and temperature function, represents a coupled complex system of second order partial differential equations (see [6]).

If the mechanical and thermal characteristics are time harmonic dependent (i.e. they are represented as the product of the time dependent exponential function $exp(-i\sigma t)$ with a complex parameter $\sigma = \sigma_1 + i\sigma_2$, $\sigma_1 \in R$, $\sigma_2 > 0$ and a function of the spatial variable $x \in \mathbb{R}^3$), then we have the so called *pseudo-oscillation equations*. The corresponding simultaneous equations generate $11 \times 11$ strongly elliptic formally non-self-adjoint matrix differential operator with constant coefficients.

The present paper is devoted to investigation of the basic boundary value problems for the system of pseudo-oscillations. First, we collect the field equations, derive the corresponding Green's identities and formulate the basic boundary value problems. Further, we construct the matrix of fundamental solutions explicitly in terms of elementary functions for the differential operator of pseudo-oscillations and establish the asymptotic properties near the origin and at infinity. Applying the potential method and the theory of singular integral equations we investigate the basic boundary value problems of pseudo-oscillations (cf. [8–13] and the references therein).

## 2. Basic differential equations

The pseudo-oscillation equations of the thermoelasticity theory of microstretch materials with microtemperatures and microdilatations in the case of isotropic homogeneous bodies according to [6] have the form

$$(\mu + \varkappa)\Delta u + (\lambda + \mu)\,\mathrm{grad\,div}\, u + \rho\sigma^2 u + \varkappa\,\mathrm{rot}\,\omega + \mu_0\,\mathrm{grad}\, v - \beta_0\,\mathrm{grad}\,\theta = -\rho H(x), \tag{2.1}$$

$$\varkappa\,\mathrm{rot}\, u + \gamma\Delta\omega + (\alpha + \beta)\,\mathrm{grad\,div}\,\omega + \delta\omega - \mu_1\,\mathrm{rot}\, w = -\rho g(x), \tag{2.2}$$

$$\varkappa_6\Delta w + (\varkappa_4 + \varkappa_5)\,\mathrm{grad\,div}\, w + \varkappa_0 w - i\sigma\mu_1\mathrm{rot}\,\omega + i\sigma\mu_2\,\mathrm{grad}\, v - \varkappa_3\,\mathrm{grad}\,\theta = \rho G(x), \tag{2.3}$$

$$-\mu_0\,\mathrm{div}\, u - \mu_2\,\mathrm{div}\, w + a_0\Delta v + \eta_0 v + \beta_1\theta = -\rho l(x), \tag{2.4}$$

$$i\beta_0 T_0\sigma\,\mathrm{div}\, u + \varkappa_1\mathrm{div}\, w + i\beta_1 T_0\sigma v + \varkappa_7\Delta\theta + i\sigma c\theta = -\rho S^*(x), \tag{2.5}$$

where $\alpha, \beta, \gamma, \lambda, \mu, \varkappa, \eta, \beta_0, \beta_1, \mu_0, \mu_1, \mu_2, a, b, a_0, b_0, \mathcal{I}, \mathcal{I}_1, \varkappa_j,\quad j = 1, 2, 3, 4, 5, 6, 7$, are the real constants characterizing the mechanical and thermal properties of the body, $\rho$ is the mass density, $\delta = \mathcal{I}_1\sigma^2 - 2\varkappa$, $\varkappa_0 = i\sigma b - \varkappa_2$, $\eta_0 = \mathcal{I}\sigma^2 - \eta$, $\sigma$ is a frequency parameter, $\sigma = \sigma_1 + i\sigma_2$, $\sigma_2 > 0$, $\sigma_1 \in \mathbb{R}$, $\Delta$ is the Laplace operator, $u = (u_1, u_2, u_3)^\top$ is the displacement vector, $\omega = (\omega_1, \omega_2, \omega_3)^\top$ is the microrotation vector, $w = (w_1, w_2, w_3)^\top$ is the microtemperature vector, $v$ is the microdilatation function, $\theta$ is the temperature, measured from a fixed absolute temperature $T_0$ $(T_0 > 0)$, $c = aT_0$; $H = (H_1, H_2, H_3)^\top$, $g = (g_1, g_2, g_3)^\top$, and $G = (G_1, G_2, G_3)^\top$ are complex-valued vector functions, connected with the body force, the body couple density, and the first heat supply moment vector, respectively; $l$ and $S^*$ are complex-valued functions connected with the external microstretch body load and the heat supply per unit mass, respectively; the superscript $(\cdot)^\top$ denotes transposition operation.

Let us introduce the matrix differential operator of order $11 \times 11$ generated by the left hand side expressions in system (2.1)–(2.5)

$$L(\partial,\sigma) := \begin{bmatrix} L^{(1)}(\partial,\sigma) & L^{(6)}(\partial,\sigma) & L^{(11)}(\partial,\sigma) & L^{(16)}(\partial,\sigma) & L^{(21)}(\partial,\sigma) \\ L^{(2)}(\partial,\sigma) & L^{(7)}(\partial,\sigma) & L^{(12)}(\partial,\sigma) & L^{(17)}(\partial,\sigma) & L^{(22)}(\partial,\sigma) \\ L^{(3)}(\partial,\sigma) & L^{(8)}(\partial,\sigma) & L^{(13)}(\partial,\sigma) & L^{(18)}(\partial,\sigma) & L^{(23)}(\partial,\sigma) \\ L^{(4)}(\partial,\sigma) & L^{(9)}(\partial,\sigma) & L^{(14)}(\partial,\sigma) & L^{(19)}(\partial,\sigma) & L^{(24)}(\partial,\sigma) \\ L^{(5)}(\partial,\sigma) & L^{(10)}(\partial,\sigma) & L^{(15)}(\partial,\sigma) & L^{(20)}(\partial,\sigma) & L^{(25)}(\partial,\sigma) \end{bmatrix}_{11\times 11}, \tag{2.6}$$

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