

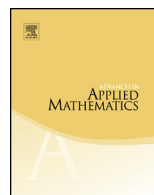


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Restricted growth function patterns and statistics <sup>☆</sup>

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## ABSTRACT

A restricted growth function (RGF) of length  $n$  is a sequence  $w = w_1 w_2 \dots w_n$  of positive integers such that  $w_1 = 1$  and  $w_i \leq 1 + \max\{w_1, \dots, w_{i-1}\}$  for  $i \geq 2$ . RGFs are of interest because they are in natural bijection with set partitions of  $\{1, 2, \dots, n\}$ . An RGF  $w$  avoids another RGF  $v$  if there is no subword of  $w$  which standardizes to  $v$ . We study the generating functions  $\sum_{w \in R_n(v)} q^{\text{st}(w)}$  where  $R_n(v)$  is the set of RGFs of length  $n$  which avoid  $v$  and  $\text{st}(w)$  is any of the four fundamental statistics on RGFs defined by Wachs and White. These generating functions exhibit interesting connections with multiset permutations, integer partitions,

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Noncrossing partition  
 Nonnesting partition  
 Pattern

and two-colored Motzkin paths, as well as noncrossing and  
 nonnesting set partitions.

rb  
 rs  
 Restricted growth function  
 Partition  
 Statistic  
 Two-colored Motzkin path

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## 1. Introduction

Recently, there has been a flurry of activity looking at the distribution of statistics over pattern classes in various objects. For example, see [4,5,7,9,11,12,14]. There are two notions of pattern containment for set partitions, one obtained by standardizing a subpartition and one obtained by standardizing a subword of the corresponding restricted growth function. In [6], the present authors studied the distribution of four fundamental statistics of Wachs and White [25] over avoidance classes using the first definition. The purpose of this paper is to carry out an analogous investigation for the second.

Let us begin by defining our terms. Consider a finite set  $S$ . A *set partition*  $\sigma$  of  $S$  is a family of nonempty subsets  $B_1, \dots, B_k$  whose disjoint union is  $S$ , written  $\sigma = B_1 / \dots / B_k \vdash S$ . The  $B_i$  are called *blocks* and we will usually suppress the set braces and commas in each block for readability. We will be particularly interested in set partitions of  $[n] := \{1, 2, \dots, n\}$  and will use the notation

$$\Pi_n = \{\sigma : \sigma \vdash [n]\}.$$

To illustrate  $\sigma = 145/2/3 \vdash [5]$ . If  $T \subseteq S$  and  $\sigma = B_1 / \dots / B_k \vdash S$  then there is a corresponding *subpartition*  $\sigma' \vdash T$  whose blocks are the nonempty intersections  $B_i \cap T$ . To continue our example, if  $T = \{2, 4, 5\}$  then we get the subpartition  $\sigma' = 2/45 \vdash T$ .

The concept of pattern is built on the standardization map. Let  $O$  be an object with labels which are positive integers. The *standardization* of  $O$ ,  $\text{stan}(O)$ , is obtained by replacing all occurrences of the smallest label in  $O$  by 1, all occurrences of the next smallest by 2, and so on. Say that  $\sigma \vdash [n]$  *contains*  $\pi$  as a *pattern* if it contains a subpartition  $\sigma'$  such that  $\text{stan}(\sigma') = \pi$ . In this case  $\sigma'$  is called an *occurrence* or *copy* of  $\pi$  in  $\sigma$ . Otherwise, we say that  $\sigma$  *avoids*  $\pi$  and let

$$\Pi_n(\pi) = \{\sigma \in \Pi_n : \sigma \text{ avoids } \pi\}.$$

In our running example,  $\sigma = 145/2/3$  contains  $\pi = 1/23$  since  $\text{stan}(2/45) = 1/23$ . But  $\sigma$  avoids  $12/3$  because if one takes any two elements from the first block of  $\sigma$  then it is impossible to find an element from another block bigger than both of them. Klazar [16–18] was the first to study this approach to set partition patterns. For more recent work, see the paper of Bloom and Saracino [3].

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