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Strongly connected synchronizing automata and the language of minimal reset words

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We approach the problem of finding strongly connected synchronizing automata with a given ideal I that serves as the set of reset words, by studying the set of minimal words M of the ideal I (no proper factor is a reset word). We first show the existence of an infinite strongly connected synchronizing automaton $\mathscr{A}$ having I as the set of reset words and such that every other strongly connected synchronizing automaton having I as the set of reset words is an homomorphic image of $\mathscr{A}$. Finally, we show that for any non-unary regular ideal I there is a strongly connected synchronizing automaton having I as the set of reset words with at most $(km^k)2^{km^k n}$ states, where k is the dimension of the alphabet, m is twice the length of a shortest word in I, and n is the number of states of the smallest automaton recognizing M. This synchronizing automaton is computable and we exhibit an algorithm to compute it in time $\mathcal{O}((k^2m^k)2^{km^k n})$.

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1. Introduction

In this paper we are interested in automata from their dynamical point of view, and not as languages recognizers. Thus, for us an automaton (for short DFA) is just a tuple $\mathscr{A} = \langle Q, \Sigma, \delta\rangle$, where Q is the set of states, Σ is the finite alphabet acting on Q, and the function $\delta : Q \times \Sigma \to Q$ describes the action of Σ on the set Q. In literature these objects are usually called *semiautomata*. We may depict an automaton as a labelled digraph having edges $q \xrightarrow{a} p$ whenever $\delta(q, a) = p$ and with the property of being complete: every $a \in \Sigma$ and $q \in Q$ there is an out-going edge $q \xrightarrow{a} p$, and being deterministic: if $q \xrightarrow{a} p$, $q \xrightarrow{a} p'$ are two edges of $\mathscr{A}$, then $p = p'$. Equivalently, a DFA may be described by an action of Σ on the set Q. Throughout the paper we will use this point of view by using the usual action-like notation $q \cdot a = \delta(q, a)$ for all $q \in Q$, $a \in \Sigma$. This action naturally extends to Σ^* and to the subsets of Q in the obvious way. Automata are mostly used in theoretical computer science as languages recognizers: by pinpointing an initial state q_0 and a set of final states $F \subseteq Q$ the automaton $\mathscr{A}$ defines the regular language $L[\mathscr{A}] = \{u \in \Sigma^* : q_0 \cdot u \in F\}$, and every regular language is recognized in this way by a finite automaton, see for instance [12,18]. The interested in automata from their dynamical point of view is mostly motivated by the longstanding Cerny's conjecture regarding the class of synchronizing automata. These are automata having a word $u \in \Sigma^*$, called *reset*, sending all the states to a unique one, i.e., $|Q \cdot u| = 1$. Cerny's conjecture states that a synchronizing automaton with n states has always a reset word of length at most $(n - 1)^2$, see [6]. The literature around Cerny's conjecture and synchronizing automata is quite impressive and span from the algorithmic point of view to the proof of Cerny's conjecture or the existence of quadratic bounds on the smallest reset word for several classes of automata, see for instance [1–4,7,9,13,24,26,28]. The best upper bound for the shortest reset word is cubic $(n^3 - n)/6$ obtained by Pin–Frankl [8,19] and recently improved by Szykuła in [25] by a factor of 4/46875. For a general survey on synchronizing automata and Cerny's conjecture see [14,27].

In this paper we continue the language theoretic approach to synchronizing automata initiated in a series of recent papers [10,11,15–17,20–23]. The starting point of such an approach is a simple observation: the set of reset words is a two-sided ideal (ideal for short) of the free monoid Σ^* that is also a regular language. The natural questions is whether any given regular ideal I may serve as the set of the reset words of some synchronizing automaton. In [15] Maslennikova has observed that the minimal DFA recognizing I is a synchronizing automaton with a sink state, i.e., a particular state s with transitions $s \xrightarrow{a} s$, for all $a \in \Sigma$, whose set of reset words is exactly I. This simple observation led Maslennikova to introduce a new notion of descriptional complexity for the class of regular ideal languages. The *reset complexity* $\mathrm{rc}(I)$ of an ideal I is the number of states of the smallest synchronizing automaton $\mathscr{B}$ for which I serves as the set of reset words of $\mathscr{B}$. The interesting fact is that Cerny's conjecture holds if and only if $\mathrm{rc}(I) \geq \sqrt{\|I\|} + 1$ with $\|I\| = \min\{|w| : w \in I\}$, holds for any ideal language I. This observation motivates the study of Cerny's conjecture and synchronizing automata