

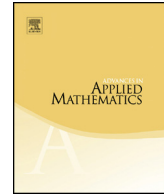


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On the Andrews–Warnaar identities for partial theta functions



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ABSTRACT

In this paper we set up a bivariate representation of partial theta functions which not only unifies some famous identities for partial theta functions due to Andrews and Warnaar but also unveils a new characteristic of such identities. As further applications, we establish a general form of Warnaar's identity and a general q -series transformation associated with Bailey pairs via the use of the power series expansion of partial theta functions.

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1. Introduction

Throughout this paper, we adopt the standard notation and terminology for q -series from the book [15] (Gasper and Rahman, 2004). As customary, the q -shifted factorials of complex variable x with the base q are given by

$$(x; q)_\infty := \prod_{n=0}^{\infty} (1 - xq^n) \quad \text{and}$$

$$(x; q)_n := \frac{(x; q)_\infty}{(xq^n; q)_\infty}$$

for all integers n . For integers $m \geq 1$, we employ the multi-parameter compact notation

$$(a_1, a_2, \dots, a_m; q)_\infty := (a_1; q)_\infty (a_2; q)_\infty \dots (a_m; q)_\infty.$$

In [15], Heine’s ${}_2\phi_1$ basic hypergeometric series with the base q and the argument x is defined as

$${}_2\phi_1 \left[\begin{matrix} a, b \\ c \end{matrix}; q, x \right] := \sum_{n=0}^{\infty} \frac{(a, b; q)_n}{(q, c; q)_n} x^n.$$

Sums of the form

$$\sum_{n=0}^{\infty} q^{An^2+Bn} x^n \quad (A > 0) \tag{1.1}$$

are called partial theta functions owing to the fact that

$$\sum_{n=-\infty}^{\infty} q^{An^2+Bn} x^n \quad (x \neq 0, A > 0)$$

is often referred to as the (complete) theta function. For the complete theta function, there holds the famous Jacobi triple product identity [15, (II.28)]

$$\sum_{n=-\infty}^{\infty} (-1)^n q^{n(n-1)/2} x^n = (q, x, q/x; q)_\infty. \tag{1.2}$$

Following [4,5], any q -series identity containing sums such as (1.1) is called a partial theta function identity. Partial theta function identities first appeared in Ramanujan’s legendary lost notebook [36], wherein he recorded many identities of such sort without any proofs. A full treatise together with a more complete bibliography on this topic can be found in [7, Chap. 6] by G.E. Andrews and B.C. Berndt. Here, we choose from [7] some typical identities as illustration.

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