

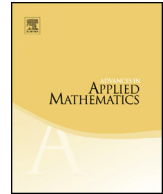


ELSEVIER

Contents lists available at ScienceDirect

Advances in Applied Mathematics

www.elsevier.com/locate/yaama



Bijjective enumerations of Γ -free 0–1 matrices

Beáta Bényi^a, Gábor V. Nagy^{b,*}^a Faculty of Water Sciences, National University of Public Service, Hungary^b Bolyai Institute, University of Szeged, Hungary

ARTICLE INFO

Article history:

Received 2 August 2017

Received in revised form 6 December 2017

Accepted 12 December 2017

MSC:

05A05

05A19

Keywords:

 Γ -free matrix

Non-ambiguous forest

ABSTRACT

We construct a new bijection between the set of $n \times k$ 0–1 matrices with no three 1's forming a Γ configuration and the set of (n, k) -Callan sequences, a simple structure counted by poly-Bernoulli numbers. We give two applications of this result: We derive the generating function of Γ -free matrices, and we give a new bijective proof for an elegant result of Aval et al. that states that the number of complete non-ambiguous forests with n leaves is equal to the number of pairs of permutations of $\{1, \dots, n\}$ with no common rise.

© 2018 Elsevier Inc. All rights reserved.

1. Introduction

We call a 0–1 matrix Γ -free if it does not contain 1's in positions such that they form a Γ configuration; i.e. two 1's in the same row and a third 1 below the left of these in the same column. $\Gamma = \begin{smallmatrix} 1 & 1 \\ 1 & * \end{smallmatrix}$. For instance, matrix A is not Γ -free because the bold 1's form a Γ configuration, while matrix B is a Γ -free matrix.

$$A = \begin{pmatrix} 0 & \mathbf{1} & \mathbf{1} & 0 \\ 1 & 1 & 0 & 0 \\ 0 & \mathbf{1} & 0 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \end{pmatrix}$$

* Corresponding author.

E-mail addresses: beata.benyi@gmail.com (B. Bényi), ngaba@math.u-szeged.hu (G.V. Nagy).

Table 1
The poly-Bernoulli numbers $B_n^{(-k)}$.

n, k	0	1	2	3	4	5
0	1	1	1	1	1	1
1	1	2	4	8	16	32
2	1	4	14	46	146	454
3	1	8	46	230	1066	4718
4	1	16	146	1066	6906	41506
5	1	32	454	4718	41506	329462

Clearly, we can say that a matrix is Γ -free if and only if it does not contain any of the submatrices from the following set:

$$\left\{ \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \right\}.$$

Pattern avoidance is an important notion in combinatorics. Matrices were also investigated from different point of view in this context; both extremal [9] and enumerative [11], [12] results are known.

Γ -free 0–1 matrices of size $n \times k$ contain at most $n + k - 1$ 1's [9]. The set of $n \times k$ 0–1 Γ -free matrices is one of the matrix classes that are enumerated by the poly-Bernoulli numbers, $B_n^{(-k)}$ [4]. Besides matrix classes that are characterized by excluded submatrices there are several other combinatorial objects that are enumerated by the poly-Bernoulli numbers. For instance, permutations with a given exceedance set, permutations with a constraint on the distance of their values and images, Callan permutations, acyclic orientations of complete bipartite graphs, non-ambiguous forests, etc. For further details, including recurrence relations and the original definition of poly-Bernoulli numbers via generating function, see [4], [5] and [6]. There is also a nice combinatorial formula of the poly-Bernoulli numbers of negative k indices: For $k > 0$,

$$B_n^{(-k)} = \sum_{m=0}^{\min(n,k)} m! \begin{Bmatrix} n+1 \\ m+1 \end{Bmatrix} m! \begin{Bmatrix} k+1 \\ m+1 \end{Bmatrix}, \quad (1)$$

where $\begin{Bmatrix} n \\ m \end{Bmatrix}$ denotes a Stirling number of the second kind. Table 1 shows the values of $B_n^{(-k)}$ for small k and n .

From (1), we give an obvious combinatorial interpretation of the numbers $B_n^{(-k)}$, which will be regarded as their combinatorial definition in this paper. (This interpretation is essentially the same as the one that counts Callan permutations.) On an (n, k) -Callan sequence we mean a sequence $(S_1, T_1), \dots, (S_m, T_m)$ for some $m \in \mathbb{N}_0$ such that $S_1, \dots, S_m$ are pairwise disjoint nonempty subsets of $\{1, \dots, n\}$, and $T_1, \dots, T_m$ are pairwise disjoint nonempty subsets of $\{1, \dots, k\}$. We note that the empty sequence is also a Callan sequence with $m = 0$.

Lemma 1. For $k > 0$, $B_n^{(-k)}$ counts the number of (n, k) -Callan sequences.

Download English Version:

<https://daneshyari.com/en/article/8900501>

Download Persian Version:

<https://daneshyari.com/article/8900501>

[Daneshyari.com](https://daneshyari.com)