

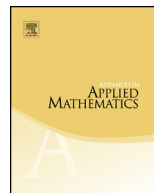


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Homogeneous q -partial difference equations and some applications

Jian Cao ^{a,b,*}

^a Department of Mathematics, Hangzhou Normal University, Hangzhou City, Zhejiang Province, 310036, PR China

^b Université de Lyon, Université Lyon 1; Institut Camille Jordan, UMR 5208 du CNRS, 43, boulevard du 11 novembre 1918, F-69622 Villeurbanne Cedex, France

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ABSTRACT

In this paper, we show how to prove identities and evaluate integrals by expanding functions in terms of products of the q -hypergeometric polynomials by homogeneous q -partial difference equations, we also generalize some results of Liu (2015) [34] and Cao (2016) [14]. In addition, we generalize multilinear and multiple generating functions for the q -hypergeometric polynomials as applications. Moreover, we deduce some recurring formulas for Ramanujan's integrals, Askey–Roy integrals, Andrews–Askey integrals and moment integrals by the method of homogeneous q -partial difference equations. Finally, we build the relation of Ismail–Zhang type generating functions for the q -hypergeometric polynomials by the method of Ismail and Zhang (2016) [26].

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* Correspondence to: Department of Mathematics, Hangzhou Normal University, Hangzhou City, Zhejiang Province, 310036, PR China.

E-mail addresses: 21caojian@gmail.com, cao@math.univ-lyon1.fr.

1. Introduction

As of today, the q -expansion formulas have been proved to be very important to the theory of basic hypergeometric series. Nevertheless, as has been pointed out by Gasper in [21] “... the succeeding higher order derivatives becomes more and more difficult to calculate for $|z| < 1$, and so one is forced to abandon this approach and to search for another way ...”.

Polynomial solutions of q -partial differential equations and q -difference equations often serve as a building block in algorithms for finding other types of closed-form solutions, computer algebra algorithms are usual tools for finding polynomials [1,4]. The motivation for this work is to deal with the q -hypergeometric polynomials as solutions of the certain q -partial difference equations. For more information, please refer to [1,4,11,25,29–35].

In this paper, we follow the notations and terminology in [22] and suppose that $0 < q < 1$. The basic hypergeometric series ${}_r\phi_s$ defined by

$${}_r\phi_s \left[\begin{matrix} a_1, a_2, \dots, a_r \\ b_1, b_2, \dots, b_s \end{matrix}; q, z \right] = \sum_{n=0}^{\infty} \frac{(a_1, a_2, \dots, a_r; q)_n}{(q, b_1, b_2, \dots, b_s; q)_n} \left[(-1)^n q^{\binom{n}{2}} \right]^{1+s-r} z^n \tag{1.1}$$

converges absolutely for all z if $r \leq s$, for $|z| < 1$ if $r = s + 1$ and for terminating. The q -series and its compact factorials are defined respectively by

$$(a; q)_0 = 1, \quad (a; q)_n = (a; q)_\infty / (aq^n; q)_\infty \tag{1.2}$$

and $(a_1, a_2, \dots, a_m; q)_n = (a_1; q)_n (a_2; q)_n \cdots (a_m; q)_n$, where $m \in \mathbb{N} := \{1, 2, 3, \dots\}$ and $n \in \mathbb{C}$. The binomial coefficient is defined by

$$\begin{bmatrix} n \\ k \end{bmatrix} = \frac{(q; q)_n}{(q; q)_k (q; q)_{n-k}}.$$

The Rogers–Szegő polynomials were introduced by Szegő in 1926 but were already studied earlier by Rogers in 1894–1895. A good definition can be found in the book by Simon [39, Ex. (1.6.5), pp. 77–87].

The homogeneous Rogers–Szegő polynomials are defined by [34, p. 3]

$$h_n(b, c|q) = \sum_{k=0}^n \begin{bmatrix} n \\ k \end{bmatrix} b^k c^{n-k} \quad \text{and} \quad g_n(b, c|q) = \sum_{k=0}^n \begin{bmatrix} n \\ k \end{bmatrix} q^{k(k-n)} b^k c^{n-k}. \tag{1.3}$$

The Al-Salam–Carlitz polynomials are defined by Al-Salam and Carlitz in 1965 [2, Eqs. (1.11) and (1.15)]

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