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Quasi-synchronization for fractional-order delayed dynamical networks with heterogeneous nodes☆

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ABSTRACT

This paper investigates the quasi-synchronization problem in heterogeneous fractional order dynamic networks with time-delay. Based on comparison theorem for the fractional order differential equation, a new fractional order functional differential inequality is built at first. According to the inequality, some quasi-synchronization conditions are derived via Lyapunov method, and the error bound is estimated. Then, the pinning control strategy is also considered via matrix analysis. Furthermore, the specific pinning schemes about how many nodes are needed to be selected are provided in an algorithm. Finally, two examples are given to verify the validity of our theoretical results.

1. Introduction

Complex dynamical networks are a lot of subsystems have been linked by edges, among which, every subsystem is formed as a dynamical system, these systems could be stable, periodic orbit or event a chaotic orbit. Many large scale systems in real world could be modeled as complex dynamical networks, such as Internet [1], multi-agent [2], social networks [3], neural networks [4], traffic networks [5] and so on. Synchronization has always been a hot topic in the researches of complex dynamical networks in last decades. Noting that there were many networks cannot be synchronized by themselves, then, some control strategies have been considered to force them reach synchronization. Some related results can be seen in [6–10].

Most of complex networks were composed of identical nodes have been studied, in which, synchronization problems have been mentioned in above results were complete synchronization. However, networks with heterogeneous nodes have been widely exist in real world. For example, for a neural network, mismatched parameters may be occur among neurons [11,12], which would cause heterogeneity for the coupled neural networks. It is worth noting that complete synchronization cannot be forced by static linear feedback controllers when heterogeneity exists. But the synchronization error may be bounded, which is called the quasi-synchronization problem. In [13], the quasi-synchronization of nonlinear coupled networks in the presence of parameter mismatches with time delay via aperiodically intermittent pinning control is investigated. More recently, the exponential synchronization issue of nonidentical coupled neural networks with time-varying

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delay is studied in [14]. There were also some other results about quasi-synchronization problems for integer order dynamical networks [15–17].

Recently, fractional order calculus has been applied for many control engineering systems [18]. Under some appropriate parameters, dynamical behavior of fractional order systems could be stability, bifurcations and even chaotic [19]. Fractional order networked systems have been gotten many results which can be see in [20–23], etc. Complex networks consist of a set of nodes with fractional order dynamics have been studied in many results. Adaptive synchronization of fractional-order complex dynamical networks with uncertain parameters has been investigated in [24]. Pinning adaptive and impulsive synchronization of fractional-order complex dynamical networks has been studied in [25]. Some other results could be found in [26–30] and references therein. However, heterogeneous dynamical networks with fractional order dynamical nodes have not been studied yet. The main reason is the lack of theoretical tools for fractional order dynamical systems.

Therefore, this paper has investigated the quasi-synchronization problem for fractional order dynamical networks, which consist of some nonidentical nodes with time-varying delay. A new fractional order differential inequality has been built at first, which would be an useful tool to analyze the quasi-synchronization problem later. Then, according to the inequality, some simple synchronization criteria have been derived. After that, pinning method has been applied based on matrix analysis techniques. An algorithm has been proposed to estimate the error bound and determine how many nodes should be selected to control next. Numerical simulations of fractional order complex networks are given to demonstrate the effectiveness of the proposed method.

Throughout this paper, I_n denotes an n dimensional identity matrix; $\lambda_{\max}(A)$ represents the maximum eigenvalue of matrix A; $\|\cdot\|$ denotes either the Euclidean vector norm or its induced matrix 2-norm and $\otimes$ denotes the Kronecker product.

2. Preliminaries and problem formulation

There were some kinds of fractional order operators. Among which, Caputo fractional operator plays an important role in the fractional systems, since the initial conditions of fractional order differential equations with Caputo derivatives take on the same form as that of integer-order differential equations, which can explain the physical meanings well. Thus, we use Caputo derivatives as main tool in this paper. Consider the following delayed fractional order chaotic systems:

$$D^{\alpha}s(t) = As(t) + Bf(s(t)) + Cg(s(t-\tau(t))), \tag{1}$$

where A, C are $\mathbb{R}^{n\times n}$ constant matrices, $\tau(t)$ is time-varying delay with bounded $0 \le \tau(t) \le \tau$. Then, consider the coupled dynamical network consists of N heterogeneous nodes, and every node has the following fractional order dynamics:

$$D^{\alpha}x_i(t) = A_ix_i(t) + B_if(x_i(t)) + C_ig(x_i(t-\tau(t))) + c\sum_{j=1}^{N}a_{ij}(x_j(t) - x_i(t)) + u_i(t),$$

where $x_i(t) = (x_{i1}(t), x_{i2}(t), \dots, x_{in}(t))^T \in \mathbb{R}^n$, A_i, B_i, C_i are $\mathbb{R}^{n\times n}$ constant matrices, $f(x_i(t)) = (f_1(x_i(t)), f_2(x_i(t)), \dots, f_n(x_i(t)))^T$ and $g(x_i(t-\tau(t))) = (g_1(x_i(t-\tau(t))), g_2(x_i(t-\tau(t))), \dots, g_n(x_i(t-\tau(t))))^T$ are nonlinear function, c is coupling strength, $u_i(t)$ is control input. $\mathcal{A} = (a_{ij})_{N\times N}$ is the adjacency matrix, the elements a_{ij} are defined as: if there is a directed path from node j to i then $a_{ij} > 0$; otherwise, $a_{ij} = 0$. The Laplacian matrix $L = (l_{ij})_{N\times N}$ is defined by: $l_{ij} = -a_{ij}$ when $i \ne j$, $l_{ii} = -\sum_{j=1, j\ne i}^{N} l_{ij}$, then the model can be rewritten as:

$$D^{\alpha}x_i(t) = A_ix_i(t) + B_if(x_i(t)) + C_ig(x_i(t-\tau(t))) - c\sum_{j=1}^{N}l_{ij}x_j(t) + u_i(t). \tag{2}$$

It is assumed that the initial conditions of network (2) and (1) are given by: $x_i(t) = \phi_i(t)$, $\tau \le t \le 0$, $i = 1, 2, \dots, N$; and $s(t) = \phi_s(t)$, $\tau \le t \le 0$, where $\phi_i(t)$ and $\phi_s(t) \in \mathcal{C}([-\tau, 0], \mathbb{R}^n)$, $\tau = \max_t \tau(t)$ and $\mathcal{C}([-\tau, 0], \mathbb{R}^n)$ is the set of continuous functions from $[-\tau, 0]$ to $\mathbb{R}^n$.

This paper considers the static linear feedback controllers, which are designed as:

$$u_i(t) = -cd_i(x_i(t) - s(t)), \;\; i = 1, 2, \dots, N,$$

where $d_i \ge 0$ are control gain. Noting that the matrices A_i, B_i, C_i implies that the network cannot be completely synchronized to the $s(t)$ by these static feedback controllers. Thus, quasi-synchronization among nodes and leader will be considered in this paper. The concept of quasi-synchronization is introduced as follows:

Definition 1 [17]. The heterogeneous dynamic network (2) is said to achieve quasi-synchronization with an error bound $\epsilon > 0$, if there exists a compact set M such that, for any $\phi_i(t)$ and $\phi_s(t) \in \mathcal{C}([-\tau, 0], \mathbb{R}^n)$, the error signal $e_i(t) = x_i(t) - s(t)$ converges into the set $M = \{e \in \mathbb{R}^n \mid \| e(t) \| \le \epsilon\}$ as $t \to +\infty$.

Noting that $s(t)$ is a chaotic system, consequently, the following assumption about $s(t)$ could be satisfied:

Assumption 1. The $s(t)$ is bounded, i.e., for any initial $\phi_s(t)$, there exists T, such that $\|s(t)\| \le \delta$, $t \ge T$, where δ is a positive constant. That is, $s(t)$ also may be an equilibrium point, a periodic orbit, or even a chaotic orbit throughout this paper.