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Wave propagation in a nonlocal diffusion epidemic model with nonlocal delayed effects

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ABSTRACT

A nonlocal diffusion epidemic model with nonlocal delayed effects is investigated. The existence and non-existence of the non-trivial and non-negative traveling wave solutions for the model are obtained, respectively. It is found that the threshold dynamics of the model is determined by the basic reproduction number of the corresponding reaction system and minimal wave speed.

1. Introduction

In this paper we study the traveling wave solutions of the following nonlocal diffusion epidemic model with nonlocal delay

$$\begin{cases} \dfrac{\partial S(x,t)}{\partial t} = d_1(J * S(x,t) - S(x,t)) - \dfrac{\beta S(x,t) K * I(x,t)}{S(x,t) + K * I(x,t) + R(x,t)}, \\ \dfrac{\partial I(x,t)}{\partial t} = d_2(J * I(x,t) - I(x,t)) + \dfrac{\beta S(x,t) K * I(x,t)}{S(x,t) + K * I(x,t) + R(x,t)} - (\gamma + \mu_1) I(x,t), \\ \dfrac{\partial R(x,t)}{\partial t} = d_3(J * R(x,t) - R(x,t)) + \gamma I(x,t) - \mu_2 R(x,t), \end{cases} \tag{1.1}$$

where

$$K * I(x,t) = \int_{-\infty}^{t} \int_{-\infty}^{\infty} K(x-y, t-s) I(y,s) dy ds$$

and

$$J * u(x,t) = \int_{-\infty}^{\infty} J(x-y) u(y,t) dy,$$

$u(x, t)$ can be either $S(x, t)$, $I(x, t)$ or $R(x, t)$ which represent the sizes of the susceptible, infected and removal individuals in location x and at time t, respectively. The positive parameters d_1, d_2 and d_3 denote the diffusion rates for the susceptible, infective and removed individuals, respectively. The constant $\beta > 0$ is the infection rate, $\gamma > 0$ refers to the removal rate

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and $\mu_i > 0$ $(i = 1, 2)$ stand for the death rates for infected and removal individuals, respectively. The convolution operator $J * S(x,t) - S(x,t)$ means that the rate of susceptible individuals in position x and at time t depends on the influence of neighboring $S(x, t)$ in all other positions y. At the same time, $J * I(x,t) - I(x,t)$ and $J * R(x,t) - R(x,t)$ describe that the rate of infected and removal individuals in position x and at time t depend on the influence of neighboring $I(x, t)$ and $R(x, t)$ in all other positions y. The kernel $K(x-y, t-s) \geq 0$ describes the interaction between the infective and susceptible individuals in location x and at the present time t which occurred in location y and at earlier time s, so the nonlocal delayed term $K^*I(x, t)$ reflects that the infected individuals have the ability of movement and infectiousness during the incubation period. Moreover, model (1.1) with standard incidence describes that some of infected and removal individuals will be removed from the population due to disease-induced death and losing-immunity-induced death, respectively, but other recovered individuals will return in the community, which captures the essential transmission dynamics. Throughout this paper, the kernel functions $J(x)$ and $K(x, t)$ satisfy the following assumptions.

(**A1**) $J(x) \in C^1(\mathbb{R}),\ J(x) = J(-x) \geq 0,\ \int_{-\infty}^{\infty} J(x)dx = 1$, J is compactly supported and r is the radius of *suppJ*.

(**A2**) $K(x, t)$ is Lipschitz continuous and compactly supported with respect to space invariable x; $K(x, t)$ is compactly supported with respect to time invariable t and the constant $T > 0$ is the length of *suppK* for invariable t.

(**A3**) $K(x, t)$ satisfies

$$\int_0^{\infty}\int_{-\infty}^{\infty} K(x,t)dxdt = 1 \quad and \quad K(x,t) = K(-x,t) \geq 0$$

for $(x,t) \in \mathbb{R} \times [0,\infty)$.

(**A4**) For every $c \geq 0$, there hold

$$\int_{-\infty}^{\infty} J(y)e^{-\lambda y}dy < \infty, \quad \int_0^{\infty}\int_{-\infty}^{\infty} K(y,s)e^{-\lambda(y+cs)}dyds < \infty, \quad \lambda \in [0,\infty)$$

and

$$\int_{-\infty}^{\infty} J(y)e^{-\lambda y}dy \to \infty, \quad \int_0^{\infty}\int_{-\infty}^{\infty} K(y,s)e^{-\lambda(y+cs)}dyds \to \infty \quad as \quad \lambda \to \infty.$$

(**A5**) $\lim\limits_{\lambda\to\infty} \frac{1}{\lambda}\int_{-\infty}^{\infty} J(y)e^{-\lambda y}dy = \infty$.

(**A6**) $\lim\limits_{\lambda\to 0^+} \frac{1}{\lambda}\int_{-\infty}^{\infty} J(y)(e^{-\lambda y} - 1)dy = 0$.

During the past several decades, a great number of theoretical issues concerning epidemic models with local and nonlocal diffusions have received considerable attention [1,3,4,15,17,24,25,29,35,38,42,43]. In particular, the existence and non-existence of the traveling wave solution to these models has been investigated extensively since these can predict whether or not the disease spread in the individuals and how fast a disease invades geographically. Recently, Wang and his coauthor [25] introduced a SIR epidemic model with Laplacian diffusion (local diffusion) and standard incidence. They obtained that their model admits a non-trivial and non-negative traveling wave solution connecting the disease-free equilibrium and the endemic equilibrium. At the same time, they showed that the model has no such traveling wave solutions.

It is known that the standard Laplacian operator corresponds to expected values for individuals moving under a Brownian process. However, the movements of individuals are often free and random which cannot be limited in a small field, so nonlocal diffusion is better described as a long range process rather than as a local one in epidemiology. During the past ten years, a special convolution operator of the form

$$J * u(x) = \int_{\mathbb{R}} J(x-y)u(y)dy$$

has been widely used to model diffusion phenomena. Yang et al. [38] replaced Laplacian operator of the model in [25] by a convolution operator and obtained the existence and non-existence of non-trivial and non-negative traveling wave solutions for their model. For other results on the nonlocal diffusion models, we refer to [5–11,16,19,20,23,26,36–43].

In real life, many diseases can not be transmitted to others immediately after being infected and have some time lag. So time delay has been suggested into many epidemic models in applications [3,4,14,15,17,33,35]. Additionally, some diseases may enhance the mobility of the infected, such as Rabies. Thus nonlocal interaction between the infected individuals and the susceptible individuals has been considered in epidemic models [15,21,24,29]. In recent years, both delay and spatial diffusion have been incorporated into deterministic models [2,12,15,18,22,27–32,43]. The nonlocal delay term involves a weighted spatio-temporal average over the whole spatial domain and the whole previous times, so it can describe the population take time to move in space and generally were not at the position in space at previous times in the model. Moreover, the researches in the last two decades showed that the existence of traveling wave solutions can indicate the spread of disease and the wave speed can illustrate the spreading speed. Based on these concerns, we suggest the nonlocal diffusion epidemic model with nonlocal (spatio-temporal) delay (1.1). The aim of the present paper is to investigate the existence and non-existence of traveling wave solutions for (1.1).

It is necessary to point out that system (1.1) has no monotone semi-flow due to the introduction of nonlocal diffusion operators. So the classical methods such as the method of monotone iteration coupled with upper-lower solutions [34] and the shooting method [13] can not be applicable any longer. Motivated by Li et al. [17,38,42], we establish the existence