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Orness measurements for lattice m-dimensional interval-valued OWA operators☆

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ABSTRACT

Ordered weighted average (OWA) operators are commonly used to aggregate information in multiple situations, such as decision making problems or image processing tasks.

The great variety of weights that can be chosen to determinate an OWA operator provides a broad family of aggregating functions, which obviously give different results in the aggregation of the same set of data.

In this paper, some possible classifications of OWA operators are suggested when they are defined on m-dimensional intervals taking values on a complete lattice satisfying certain local conditions. A first classification is obtained by means of a quantitative orness measure that gives the proximity of each OWA to the OR operator. In the case in which the lattice is finite, another classification is obtained by means of a qualitative orness measure. In the present paper, several theoretical results are obtained in order to perform this qualitative value for each OWA operator.

1. Introduction

Interval-valued fuzzy sets have shown to be a good tool for modeling some situations in which uncertainty is present [2]. This class of fuzzy sets allows us to assign a whole interval to each element of the set, which is more flexible than a single value to represent the reality. However, the aggregation of intervals, necessary in most decision making problems or image processing techniques in order to obtain a global value from several data, is not always an easy task.

Ordered weighted average (OWA) operators are commonly used when the fuzzy sets that are involved in this case of problems take single real values instead of intervals [5,13,14]. These aggregation functions, introduced by Yager [11], merge the data after modulating them by means of some weights, but in such a way that the weight affecting to each datum only depends on the place it takes in the descending chain of the arranged data. Hence Yager's OWA operators are symmetric,

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i.e., the global value that they obtain from a collection of data does not depend on either the expert or the resource that has provided each datum.

One of the advantages of OWA operators is their flexibility. The different weighting vectors provide a broad family of aggregation functions, varying from an OR aggregation (maximum) to an AND aggregation (minimum). One of the most difficult tasks for using OWA operators is the choice of its weighting vector. For this reason, Yager gives a classification of OWA operators by assigning an orness measure to each one of them. This value gives an idea of the proximity of each OWA operator to the OR one. Specifically, orness yields the maximum value (1) to the OR operator while it yields the minimum value (0) to the AND one.

Similar to other aggregation functions, see [6], OWA operators were generalized by Lizasoain and Moreno [7] from the real unit interval to a general complete lattice L endowed with a t-norm T and a t-conorm S, whenever the weighting vector satisfies a distributivity condition with respect to T and S. Moreover, a qualitative parameterization of OWA operators, based on their proximity to the OR operator, but only in those cases in which the lattice L is finite, is studied in [9].

In [10], a quantitative parameterization of OWA operators is proposed for a wider family of lattices L: those containing a Maximal Finite Chain between any two elements. These lattices have been referred to as (MFC)-lattices and they comprise in particular all the finite lattices. The quantitative orness on these (MFC) lattices is defined in the following way:

First, for each weighting vector $\alpha=(\alpha_1,\dots,\alpha_n)\in L^n$, a qualitative quantifier $Q:\{0,1,\dots,n\}\to L$ is defined by means of $Q_\alpha(0)=0_L$ and $Q_\alpha(k)=S(\alpha_1,\dots,\alpha_k)$ for $1\le k\le n$.

Then, instead of merging the weights as it had been done by Yager in the real case, the formula for the orness of OWA operators on lattices considers, for each $k\in\{1,\dots,m\}$, the length of the shortest maximal chain $\mu(k)$ between $Q_\alpha(k-1)$ and $Q_\alpha(k)$. Then it aggregates them according to Yager's formula:

$$\mathrm{orness}(F_\alpha)=\frac{1}{n-1}\sum_{k=1}^{n}(n-k)\frac{\mu(k)}{\mu(1)+\cdots+\mu(m)}.$$

The present paper is devoted to the classification of m-dimensional interval-valued OWA operators. It deals with OWA operators defined on the lattice L^{I_m} comprising all m-dimensional intervals $[a_1,\dots,a_m]$ with $a_1\le_L\cdots\le_L a_m$ belonging to a lattice L. The name of m-dimensional interval responds to the following reasons.

In the context of real-valued fuzzy sets, binary intervals are commonly used to express the membership degree of an element to a fuzzy set when some uncertainty or noise is present. As a generalization of them, m-dimensional real intervals are introduced in [1] to express membership degrees given by m different evaluation processes ordered by rigidity. For a general complete lattice L, m-dimensional intervals are studied in [8].

In the present paper, the case in which L is an (MFC)-lattice is considered. In particular, it is shown that, in that case, L^{I_m} is also an (MFC)-lattice. The quantitative orness defined in [10] for each lattice m-dimensional interval OWA operator is performed as a weighted average of the orness measures carried out componentwise. When $m=1$, the results obtained here agree with those of [10].

In a complementary way, in those cases in which L is finite, it is obvious that L^{I_m} is also finite and the qualitative orness given in [9] is well-defined on this new lattice. However, the calculation of the elements belonging to L^{I_m} that occurs in the qualitative orness formula is not an easy task. We have achieved a formula for these elements in two common cases of L: when L is a distributive and complemented finite lattice and when L is a finite chain, which are shown in several examples of decision making problems. Also in this case, the results when $m=1$ agree with those of [9].

The remainder of the paper is organized as follows. Section 2 provides some preliminary concepts and results regarding OWA operators defined on a complete lattice. We show that, if a lattice L satisfies some local finiteness condition, then so does L^{I_m}, in Section 3. In this section, we also study the relationship between the quantitative L^{I_m}-orness and the quantitative L-orness of OWA operators and we apply it to a decision making problem. In Section 4, we consider the particular case of finite lattices as well as we shown how to find the elements in L^{I_m} that are necessary to calculate the qualitative orness of OWA operators defined on it. In Section 5, we analyze those cases in which the finite lattice is distributive and complemented and show an application of this modelization to a decision making problem. We study those cases in which the lattice is a finite chain and apply the results to some decision making problems in Section 6. We finishes with some conclusions.

2. Preliminaries

Throughout this paper $(L,\le_L)$ will denote a complete lattice, i.e., a partially ordered set, finite or infinite, for which all subsets have both a supremum (least upper bound) and an infimum (greatest lower bound). We denote 0_L and 1_L, respectively, as the least and the greatest elements in L.

Recall that an n-ary aggregation function is a function M: $L^n\to L$ such that:

(i) $M(a_1,\dots,a_n)\le_L M(a'_1,\dots,a'_n)$ whenever $a_i\le_L a'_i$ for $1\le i\le n$.
(ii) $M(0_L,\dots,0_L)=0_L$ and $M(1_L,\dots,1_L)=1_L$.