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Mean-square dissipative methods for stochastic age-dependent capital system with fractional Brownian motion and jumps

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ABSTRACT

In this paper, we analyze mean-square dissipativity of numerical methods applied to a class of stochastic age-dependent (vintage) capital system with fractional Brownian motion (fBm) and Poisson jumps. Some sufficient conditions are obtained for ensuring the underlying systems are mean-square dissipative. It is shown that the mean-square dissipativity is preserved by the compensated split-step backward Euler method and compensated backward Euler method without any restriction on stepsize, while the split-step backward Euler method and backward Euler method could reproduce mean-square dissipativity under a stepsize constraint. Those results indicate that compensated numerical methods achieve superiority over non-compensated numerical methods in terms of mean-square dissipativity. A numerical example is provided to illustrate the theoretical results.

1. Introduction

In the past few years, the vintage capital model described by ordinary differential equations has widely received a great deal of attention among economists, because it provides an appealing framework for the analysis of investment volatility [3,5,9,25,26]. For example, Feichtinger et al. [4] developed the vintage capital stock model under technological progress, while the model is solved. Goetz et al. [9] studied the capital replacement decision of a firm as a distributed investment and disinvestment optimal control problem. Zhang [25] discussed the convergence of numerical solutions for a class of stochastic age-dependent capital system with Markovian switching. By analyzing these models of age-dependent (vintage) capital, we are able to realize that economy growth model concerns four variables: output, capital, labor, and technological progress. Capital, labor and technological progress are combined to produce output. However, some important sources of uncertainty may be discontinuous and fluctuating. To make the vintage capital model more realistic than previous model, we add the fractional Brownian motion (fBm) and jumps to describe the stochastic burst phenomena.

Dissipativity is one of the most important issues in the study of dynamical systems, introduced in the 1970s, generalizes the idea of a Lyapunov stability. There are applications in such areas as stability theory, synchroization control, system norm estimation, and robust control. Therefore, it is a very interesting topic to investigate the dissipativity of dynamical systems.

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They are characterized by giving a bounded positively invariant absorbing set that all trajectories starting from any bounded set enter in a finite time and thereafter remain inside.

On the other hand, for previous work on the dissipativity of numerical methods for deterministic ordinary differential equations, we refer to [12,14]. Many authors attempt to explore the dissipativity of different numerical methods for deterministic functional differential equations [6,21,24]. Recently, Huang [13] analyzed the mean-square stability and dissipativity of two classes of θ methods for systems of stochastic delay differential equations. Ma et al. [18] introduced mean-square dissipativity of several numerical methods for stochastic differential equations with jumps. However, few studies endeavor to look into the mean-square dissipativity of numerical methods applied to stochastic age-dependent capital system with fractional Brownian motion and jumps.

Moreover, most of the stochastic age-dependent capital system, similar to the stochastic differential equations, Thus, numerical approximation schemes are invaluable tools for exploring its properties. For example, Higham and Kloeden [10] discussed the split-step backward Euler method and compensated split-step backward Euler method for stochastic differential equations with jumps. Higham and Kloeden [11] investigated the convergence and stability of stochastic θ method for jump-diffusion systems. Wang and Gan [23] studied compensated stochastic θ methods for stochastic differential equations with jumps. We can also find other results on numerical methods for stochastic differential equations [1,2,7,15–17,19,20,22].

In this paper, we consider a class of stochastic age-dependent capital system with fractional Brownian motion (fBm) and Poisson jumps of form

$$\begin{cases} d_tK(t,a) = \left[-\frac{\partial K(t,a)}{\partial a} - \mu(t,a)K(t,a) + f(t,K(t,a)) \right]dt \\ \qquad +g(t,K(t,a))dB^H(t) + h(t,K(t,a))dN(t), & \text{in } J, \\ K(t,0) = \phi(t) = \gamma(t)A(t)F(L(t), \int_0^A K(t,a)da), & \text{in } t\in[0,T], \\ K(0,a) = K_0(a), & \text{in } a\in[0,A], \\ N_1(t) = \int_0^A K(t,a)da, & \text{in } t\in[0,T], \end{cases} \tag{1.1}$$

where $K_0(a){:=}K_0$ is a random variable and $E|K_0|^2 < +\infty$. $J=(0,A)\times(0,T)$, $d_tK(t,a) := \frac{\partial K(t,a)}{\partial a}dt$, $f(t,\cdot)$, $h(t,\cdot): L_U^2 \to U$, $g(t,\cdot): L_U^2 \to \mathscr{L}_2^0(M,U)$. Here $\mathscr{L}_2^0(M,U)$ denotes the space of all Q-Hilbert-Schmidt operators from M into U (see Section 2). The stock of capital goods of age a at time t is denoted by $K(a, t)$, $N_1(t)$ is the total sum of the capital, a is the age of the capital, the investment $\phi(t)$ in the new capital, and the investment $f(t, K(a, t))$, in the capital of age a are the endogenous (unknown) variables. The maximum physical lifetime of capital A, the planning interval of calendar time $[0, T]$, the depreciation rate $\mu(a, t)$ of capital, and the capital density $K_0(a)$ (the initial distribution of capital over age) are given. The production function $F(L(t), \int_0^A K(t,a))$ is neoclassical, where $\int_0^A K(t,a)$ is the total sum of the capital and $L(t)$ is the labor force. $\gamma(t)$ denotes the accumulative rate at the moment of t; $0<\gamma(t)<1$, and $A(t)$ is the technical progress at the moment of t. $f(t,K(a,t)) + g(t,K(a,t))\frac{dB^H(t)}{dt} + h(t,K(a,t))\frac{dN(t)}{dt}$ denotes effects of external environment for capital system, such as innovations in technique, introduction of new products, natural disasters, and changes in laws and government policies, and so on. $h(t, K(a, t))$ is a jump coefficient, $B^H(t)$ is an fractional Brownian motion (fBm) with Hurst parameter $H\in(\frac{1}{2},1)$ and $N(t)$ is a scalar Poisson process with intensity $\lambda>0$. We assume that Poisson process $N(t)$ is independent of the fBm $B^H(t)$. The (1.1) is a generalization of the stochastic capital equation. It describes the evolution of the composition of the productive capital as a function of purchasing /selling new or used capital.

A new stochastic age-dependent (vintage) capital system is given by model (1.1). It is an extension of Zhang [25] and Zhang et al. [26]. The effects of the stochastic environmental noise considerations lead to a stochastic age-dependent capital system (1.1), which is more realistic.

A is the maximal age of the capital, so

$$K(t) := K(t,a) = 0, \quad \forall a \geq A.$$

Motivated by the above description, in this paper we will focus on the mean-square dissipativity of numerical methods for a stochastic age-dependent capital system with fBm and Poisson jumps under the given conditions. Our work differs from these references [25] and Zhang et al. [26] in that (a) compensated numerical methods and non-compensated numerical methods are considered, and (b) mean-square dissipativity and fBm are involved.

The main purpose of this paper is to investigate the mean-square dissipativity of numerical methods applied to stochastic age-dependent capital system with fBm and jumps. In Section 2, we introduce some notations, definitions and assumptions, which will be used throughout the rest of the paper. In Section 3, Some criteria for mean-square dissipative of (1.1) is shown. And then the mean-square dissipativity of split-step backward Euler method and compensated split-step backward Euler method for (1.1) are established in Section 4. Section 5 is devoted to prove that backward Euler method and compensated backward Euler method can inherit the mean-square dissipativity of (1.1). An example is given to illustrate our theory in Section 6.