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Numerical algorithm for time-fractional Sawada-Kotera equation and Ito equation with Bernstein polynomials

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ABSTRACT

The generalized KdV equation arises in many problems in mathematical physics. In this paper, an effective numerical method is proposed to solve two types of time-fractional generalized fifth-order KdV equations, the time-fractional Sawada-Kotera equation and Ito equation, the idea is to use Bernstein polynomials. Firstly, Bernstein basis polynomials are utilized to approximate unknown function and the error bound is given. Secondly, the representation of Bernstein basis polynomials are proposed to easily and quickly obtain the integer and fractional differential operator of unknown function, by which the studied equations can be displayed as the combination of operator matrices. Finally, comparison with Chebyshev wavelets method and the error data are presented to demonstrate the high accuracy and efficiency of Bernstein polynomials method for this kind of wave equations.

1. Introduction

In resent years, fractional calculus has been paid considerable attention due to its important applications in diverse and widespread fields of engineering and science. Various important phenomena in physics, electromagnetics, viscoelasticity, fluid mechanics, electrochemistry, advection-diffusion models, biological population models, optics and signals processing can be well described by fractional differential equations [1]. For that reason the research of the solutions of fractional differential equations has become a hot spot. However, since the kernel is fractional, it is difficult to obtain the analytical solutions. Therefore, many numerical methods have been developed for fractional differential equations, such as q-Homotopy Analysis Method [2,3], Wavelets Method [4], Adomian Decomposition Method [5], Finite Difference Method [6], etc.

Nonlinear wave phenomena exist in many fields such as fluid mechanics, plasma, physics, biology, hydrodynamics, solid state physics and optical fibers [7–11]. The Korteweg-de Vries (KdV) equation is an evolution equation proposed as a model for one-dimensional long surface waves of water in a narrow and shallow channel, as well as in other homogeneous, weakly nonlinear and weakly dispersive media [12]. The generalized KdV equation as the development of the KdV equation is an essential model for several physical phenomena [13–16]. Therefore, it is considered particularly important in mathematical physics. The currently proposed numerical methods for these equations are Chebyshev wavelet method [1], hybrid LDG-HWENO scheme [17], Modified Adomian Decomposition Method [18], Homotopy Analysis Method [19] and q-Homotopy Analysis method [20], which have never been considered by polynomials method.

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In this paper, we use polynomials method to solve the following time-fractional generalized fifth-order KdV equation numerically:

$$\frac{\partial^\alpha u}{\partial t^\alpha} + au^2\frac{\partial u}{\partial x} + b\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial x^2} + cu\frac{\partial^3 u}{\partial x^3} + d\frac{\partial^5 u}{\partial x^5} = 0, \quad 0 < \alpha \le 1. \tag{1.1}$$

Taking $a = 45, b = 15, c = 15, d = 1,$ we obtain the time-fractional Sawada-Kotera equation [1,20]:

$$\frac{\partial^\alpha u}{\partial t^\alpha} + 45u^2\frac{\partial u}{\partial x} + 15\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial x^2} + 15u\frac{\partial^3 u}{\partial x^3} + \frac{\partial^5 u}{\partial x^5} = 0, \quad 0 < \alpha \le 1. \tag{1.2}$$

Taking $a = 2, b = 6, c = 3, d = 1,$ we obtain the time-fractional Ito equation [20]:

$$\frac{\partial^\alpha u}{\partial t^\alpha} + 2u^2\frac{\partial u}{\partial x} + 6\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial x^2} + 3u\frac{\partial^3 u}{\partial x^3} + \frac{\partial^5 u}{\partial x^5} = 0, \quad 0 < \alpha \le 1. \tag{1.3}$$

2. Preliminaries

In this section, we review some essential definitions and mathematical preliminaries about fractional calculus which are applied in the next sections. The most used fractional calculus definitions are Riemann–Liouville's and Caputo's definitions, Caputo fractional derivative is a modification of the Riemann–Liouville's definition. In this paper, the definition of Caputo's sense is considered due to its advantage of dealing with integer order initial conditions for fractional order differential equations.

Definition 1: The Riemann–Liouville fractional derivative operator D^α of order α is defined as [21]

$$D^\alpha u(x) = \begin{cases} \dfrac{1}{\Gamma(m-\alpha)}\dfrac{d^m}{dx^m}\int_0^x (x-\tau)^{\alpha-m+1}u(\tau)d\tau, & \alpha > 0,\ m-1 \le \alpha < m, \\ \dfrac{d^{(m)}u(x)}{dx^m}, & \alpha = m. \end{cases} \tag{2.1}$$

Definition 2: The Caputo fractional derivative operator D^α of order α is defined as [21,22]:

$$D^\alpha u(x) = \begin{cases} \dfrac{1}{\Gamma(m-\alpha)}\int_0^x (x-\tau)^{\alpha-m+1}u^{(m)}(\tau)d\tau, & \alpha > 0,\ m-1 \le \alpha < m, \\ \dfrac{d^{(m)}u(x)}{dx^m}, & \alpha = m, \end{cases} \tag{2.2}$$

where $x \ge 0$, and $m \in \mathbb{N}$ ($\mathbb{N}$ denote positive integers). In this paper, $0 < \alpha \le 1$ is considered.

For $0 < \alpha \le 1$ and constant C, Caputo fractional derivative have some most important properties:

$$\mathcal{D}^\alpha C = 0. \tag{2.3}$$

$$\mathcal{D}^\alpha(\lambda u(x)) = \lambda\mathcal{D}^\alpha u(x). \tag{2.4}$$

$$\mathcal{D}^\alpha x^m = \begin{cases} 0, & m = 0, \\ \dfrac{\Gamma(m+1)}{\Gamma(m+1-\alpha)}x^{m-\alpha}, & m = 1, 2, 3, \ldots. \end{cases} \tag{2.5}$$

3. Function approximation using Bernstein polynomials and error analysis

3.1. Bernstein polynomials

The Bernstein basis polynomials of degree n in [0,1] are defined as [23,24]:

$$\mathcal{B}_{n,i}(x) = \binom{n}{i}x^i(1-x)^{n-i} = \sum_{k=0}^{n-i}(-1)^k\binom{n}{i}\binom{n-i}{k}x^{i+k}, \tag{3.1}$$

where $i = 0, 1, 2, \ldots, n$. Then, we define