Applied Mathematics and Computation 338 (2018) 12–24

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Traveling waves of some Holling–Tanner predator–prey system with nonlocal diffusion

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ARTICLE INFO

Keywords:
Traveling waves
Predator–prey model
Nonlocal diffusion
Schauder's fixed point theorem
Coexistence state

ABSTRACT

This paper is devoted to establish the existence and non-existence of the traveling waves for the nonlocal Holling–Tanner predator–prey model. By applying the Schauder's fixed point theorem, we can obtain the existence of the traveling waves. Moreover, in order to prove the limit behavior of the traveling waves at infinity, we construct a sequence that converges to the coexistence state. For the proof of the nonexistence of the traveling waves, we use the property of the two-sided Laplace transform. Finally, we give the effect of the nonlocal diffusion term for the traveling waves.

1. Introduction

In recent years, the dynamic relationship between predators and their prey has been one of the dominant themes in ecology due to its universal existence and importance. Researchers have established many models due to the different functional response to predation, see [22–24]. The classical Lotka–Volterra model and its modified models have been studied for the stability of equilibria and the existence of traveling waves, see [6,14,17,20,28–30,33] et al. The properties of the model with the Leslie–Gower functional response and its modified models have been discussed, for instance [1,21,25,36], et al. For the predator–prey model

$$\begin{cases} \dfrac{du(t)}{dt} = u(t)(1-u(t)) - \Pi(u(t))v(t), \\ \dfrac{dv(t)}{dt} = rv(t)\left(1-\dfrac{v(t)}{u(t)}\right), \end{cases} \tag{1.1}$$

the second equation means that the intrinsic population growth rate r affects not only the potential increase of the population but also its decrease. If v is greater than u, the population will decline, and the speed of its decline is directly proportional to the intrinsic growth rate. It seems to be a contradiction, but it is realistic since that species of small body size and early maturity have high intrinsic growth rates and also have low survival rates and short lives. Typical example of the functional response $\Pi(u)$ is given by Holling-type functional response in [15]. Hsu and Huang [16] have studied the global stability property with three types of functional responses. Tanner [31] has considered the stability of the system

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https://doi.org/10.1016/j.amc.2018.04.049

(1.1) with $\Pi(u) = \dfrac{mu}{A+u}$ $(m>0,\ A>0)$. A. Ducrot [11] has studied the predator-prey model with the Laplacian diffusion

$$\begin{cases} \dfrac{\partial}{\partial t}u(x,t) = d\Delta u(x,t) + u(x,t)(1-u(x,t)) - \Pi(u(x,t))v(x,t), \\ \dfrac{\partial}{\partial t}v(x,t) = \Delta v(x,t) + rv(x,t)\left(1 - \dfrac{v(x,t)}{u(x,t)}\right), \end{cases} \tag{1.2}$$

where $d>0$ describes the diffusivity of prey, r denotes the growth rate of predator and $\Pi(u) = u\pi(u)$, π: $[0,\infty) \to [0,\infty)$ is of the class C^1 such that $\pi(u)>0$ for all $u \in (0,1]$. He has proved that the system has a spreading speed property and the solution converges towards a generalized transition wave with some determined global mean speed of propagation for the one-dimensional system.

Although the standard Laplacian operator can described the movement of individuals under a Brownian process, the movement of individuals which cannot be limited in a small area is often free and random. So recently, various integral operators have been widely used to model the nonlocal diffusion phenomena. For example, an operator of the form

$$K[u](x) = \int_{\mathbb{R}} k(x,y)(u(y) - u(x))dy$$

appears in the theory of phase transition, ecology, genetics and neurology, see [2,18,19,32]. Meanwhile, many researchers give more attention on the study of traveling waves of nonlocal reaction diffusion equations. For instance, many authors have obtained the properties of the solution of the reaction–diffusion systems with nonlocal diffusion term, see [3,4,7–10,27,37,38]. In [26], the authors have obtained the existence of the traveling waves of the model without monotone condition. In [5], we have considered the spreading speed properties for the model (1.2) with the fractional diffusion term $\Delta^\alpha (\alpha \in (0,1))$.

In the present paper, we consider the Holling–Tanner predator-prey model with the classical Lotka–Volterra functional response, that is the following model

$$\begin{cases} \dfrac{\partial}{\partial t}u(x,t) = d_1(J * u(x,t) - u(x,t)) + u(x,t)(1-u(x,t)) - \beta u(x,t)v(x,t), \\ \dfrac{\partial}{\partial t}v(x,t) = d_2(J * v(x,t) - v(x,t)) + rv(x,t)\left(1 - \dfrac{v(x,t)}{u(x,t)}\right), \end{cases} \tag{1.3}$$

where $d_i>0$ $(i=1,2)$ are diffusion rates for the prey and predator individuals, respectively, $J * u(x,t) = \int_{\mathbb{R}} J(y)u(x-y,t)dy$ and $J * v(x,t) = \int_{\mathbb{R}} J(y)v(x-y,t)dy$ represent the standard convolutions with space invariable x, βu $(0<\beta<1)$ denotes the functional response to predation, and $r>0$ denotes the growth rate of predator. Throughout this paper, we need the below assumptions of the diffusion kernel J.

Assumption 1.1. (J_1) J is a smooth function in $\mathbb{R}$ and satisfies $J \in C^1(\mathbb{R})$, $J(x) = J(-x) \geq 0$, $\int_{\mathbb{R}} J(x)dx = 1$.
(J_2) There exists $\lambda_0 \in (0,+\infty]$ such that $\int_{\mathbb{R}} J(x)e^{-\lambda x}dx < +\infty$ for any $\lambda \in [0,\lambda_0)$, and $\int_{\mathbb{R}} J(x)e^{-\lambda x}dx \to +\infty$ as $\lambda \to \lambda_0 - 0$.

In this work, we mainly study the existence and nonexistence of traveling waves which connect the predator free state (1,0) with the coexistence state $(\frac{1}{1+\beta}, \frac{1}{1+\beta})$ for the system (1.3). We will obtain that there exists a critical velocity $c^*>0$ such that for $c>c^*$, the system (1.3) admits a traveling wave solution with wave speed c; for $0<c<c^*$, the system (1.3) has no traveling waves with wave speed c. Further, we can deduce that the existence of the traveling waves is independent of spatial movement patterns of the predator and prey. The spreading speed of the traveling waves, however, depends on the movement of the predator. By our analysis, we can confirm that the diffusion rate d_2 and the growth rate r of the predator can increase the spreading speed. The main approach for the proof of the existence is to construct a suitable invariant set and then to use the Schauder's fixed point theorem, see [12,13,26,27]. To overcome the difficulties of the nonlocal diffusion, we construct an invariant cone in a large bounded domain and then pass to the unbounded domain. Finally, we conclude the nonexistence of the traveling waves with speed $c<c^*$ by using the two-sided Laplace transform.

This paper is organized as follows. In the next section, we will give some preliminaries for the proof of our results. In Section 3, we present the proof of the existence of the traveling waves by Schauder's fixed point theorem under the assumption of the compact support for the kernel function J. In Section 4, we obtain the nonexistence of traveling waves by two-sided Laplace transform. Finally, we give some discussion about the effect of nonlocal diffusion.

2. Some preliminaries

In this section, we will give some useful results for the proof of the existence and nonexistence of the traveling waves for the system (1.3). In the sequel, we always assume that the initial equilibrium is (1,0). The traveling wave solution means