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Dynamical behaviors analysis of memristor-based fractional-order complex-valued neural networks with time delay

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ABSTRACT

The robust stability for memristor-based fractional-order Complex-Valued Neural Networks (FCVNNs) with time delay is investigated here. In complex plane, by using the Lyapunov method, and under the sense of Filippov solutions, the existence of unique equilibrium and globally asymptotic stability for such Neural Networks (NNs) have been obtained when the nonlinear complex-valued activation functions could be split into two(real and imaginary) parts. Moreover, locally asymptotic stability for such Neural NNs have been proposed when the nonlinear complex activation functions are bounded. Lastly, three numerical examples are given to confirm the efficiency of theorems.

1. Introduction

As the extension of the conventional integer-order calculus, fractional-order calculus has many applications in the nonlinear fields. Because the fractional-order derivative is a pretty good tool to describe the memory and hereditary properties in diverse processes. And fractional-order differential equations provided a valuable instrument in the modeling of many systems in various domains, such as engineering and science [2,3]. In addition, due to the infinite memory of fractional-order systems, they can be taken to neural network systems, which is an extremely important improvement.

The studying of NNs has a particular meaning for understanding many vital processes in various areas [4,5]. So far, many studies about stability analysis of NNs have been acquired [6–10]. At present, the research of dynamical behaviors of Complex-Valued Neural Networks(CVNNs) can be found in many researches. Due to the complex signals have been involved in the applications of NNs, and also for the extension of Real-Valued Neural Networks(RVNNs), CVNNs include complex-valued information in the complex plane. Compared with the RVNNs, CVNNs' behaviors and varieties are more complicated. Besides, activation functions are either bounded or analytic based on the Liouville's theorem in the CVNNs, which means that finding a suitable activation function for CVNNs is a hard work. Therefore, being different from RVNNs, different kinds of activation functions need to be taken in complex valued neural networks. Some results have already been obtained [11–13]. In Ref. [11], the issue of global stability of the CVNNs with time-delays was studied. The Ref. [12] studies uniform stability of FCVNNs with time delay and discussed the existence. However, none of the papers took into account the impact of the memristor.

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In 1971, memristor as the fourth basic passive circuit element was first raised by Chua [14,15]. What's more, it shares lots of properties of resistors and offers a nonvolatile memory storage within a simple device configuration. Then it has attracted more and more attentions due to its property of memorizing its history and displaying features of pinched hysteresis. Based on these special properties, the memristor has been applied in NNs and the memristor-based NNs were firstly introduced in Ref. [16]. In addition, many people have studied the model of memristor-based NNs in the real area the last few years [17–20]. But in complex plane, there are a few results, such as the discussion about finite-time stability analysis, which system was memristor-based FCVNNs with time delays [21], and different from the previous one, both leakage and time-varying delays were considered [22], but they are all about finite-time stability analysis.

Because the phenomenon of deviations and perturbations in parameters, some uncertainties have existed in lots of physical and engineering systems unavoidably [23,24]. Particularly, uncertainties have great impact on NNs, because the connection weights are contingent on certain capacitance and resistence values that comprise errors or uncertainties [25]. Therefore, it is necessary to ensure whether the neural network is stable with parameters uncertainties and to estimate the uncertainties.

For the reason above, the memristor-based FCVNNs with time delay are introduced in this paper, and robust stability of CVNNs is studied. And the mainly contributions are three new theories obtained as follows. First, by given assumptions, the unique of the existing equilibrium point is analyzed by using homomorphism theory. Second, globally robust stability of such neural system is analyzed via a Lyapunov function when the activation function could be divided into its real and imaginary part. Moreover, locally robust stability is studied when the activation function could not be separated. In addition, numerical simulations are proposed to show efficiency of theorems.

The rest hereof is organized as below. The FCVNN model representation and some preliminaries are presented in Section 2. Stability analysis of memristor-based FCVNNs with parameter uncertainties and time delay are represented in Section 3. Numerical examples are represented in Section 4. Finally, some outcomes are obtained in Section 5.

Notions: Throughout this paper, let $R=(-\infty,+\infty)$, $R^n(C^n)$ be the n-dimensional Euclidean space(unitary space). $x^T(A^T)$ is the transpose of vector x(matrix A). For any $x\in C$, $\bar{x}$ denotes the conjugate of x. For a square real matrix A, $[A]^s$ denotes $[A]^s=\frac{A+A^T}{2}$.

2. Model description and preliminaries

The memeristor-based NNs with time delay under the Filippov sense has been constructed. In addition, the Caputo fractional-order derivative is used to describe our model.

Definition 1. (Caputo fractional-order derivative [1]) The Caputo fractional-order derivative is

$$ {}_{t_0}D_t^{\alpha} f(t)=\frac{1}{\Gamma(n-\alpha)}\int_{t_0}^{t}\frac{f^{(n)}(\tau)}{(t-\tau)^{\alpha+1-n}}d\tau, $$

where $\alpha>0$, $f(t)\in C^n([t_0,+\infty])$, and $\Gamma(\cdot)$ is Gamma function, n is a positive integer satisfied $n-1<\alpha\le n$.

Remark 1. a and b are constants, and the fractional-order systems have the following property:

$$ {}_0D_t^{\alpha}(ax(t)+by(t))=a_0D_t^{\alpha}x(t)+b_0D_t^{\alpha}y(t). $$

Definition 2. (Filippov [26]) $\frac{dx}{dt}=f(t,x)$ is a differential system, and $f(t, x)$ is discontinuous in x. The set-valued maps of $f(t, x)$ is

$$ F(t,x)=\bigcap_{\delta>0}\bigcap_{\mu(N)=0} co[f(B(x,\delta)\setminus N)], $$

$B(x,\delta)=\{y:||y-x||\le\delta\}$ is a ball with a radius of δ, and as the center of x, where $\mu(N)$ is the Lebesgue measure of set N. A Filippov solution of system (1) with the condition $x(0)=x_0$ is absolutely continuous on $[t_1, t_2]\subsetneqq[0, T]$, and satisfies $\frac{dx}{dt}\in F(t,x)$, for a.e. $t\in[0, T]$.

The Filippov definition for fractional-order differential system ${}_{t_0}D_t^{\alpha}x(t)=f(t,x)$, with the condition $x(0)=x_0$, $0<\alpha<1$, we also could get the similar results.

Definition 3. [27] $A=(a_{ij})_{n\times n}$ is a matrix that all off-diagonal elements are non positive. If one of the following conditions holds,

1) All principal minors of A are positive;
2) A has all positive diagonal elements and there exists a positive diagonal matrix $\Lambda=diag(\lambda_1,\lambda_2,\dots,\lambda_n)$ such that $A\Lambda$ is strictly diagonally dominant, that is

$$ a_{ii}\lambda_i>\sum_{j\neq i}\lambda_j,\quad i=1,2,\dots,n, $$

which can be written as

$$ \sum_{j=1}^{n}\lambda_j>0,\quad i=1,2,\dots,n; $$