Applied Mathematics and Computation 339 (2018) 272–285

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Numerical solution of three-dimensional Volterra–Fredholm integral equations of the first and second kinds based on Bernstein's approximation

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ARTICLE INFO

MSC:
65R20
41A50
41A55

Keywords:
Volterra–Fredholm integral equations
Numerical method
Three-variable Bernstein polynomials
Error estimation

ABSTRACT

A new and efficient method is presented for solving three-dimensional Volterra–Fredholm integral equations of the second kind (3D-VFIEK2), first kind (3D-VFIEK1) and even singular type of these equations. Here, we discuss three-variable Bernstein polynomials and their properties. This method has several advantages in reducing computational burden with good degree of accuracy. Furthermore, we obtain an error bound for this method. Finally, this method is applied to five examples to illustrate the accuracy and implementation of the method and this method is compared to already present methods. Numerical results show that the new method provides more efficient results in comparison with other methods.

1. Introduction

The multi-dimensional Volterra–Fredholm integral equations can be arisen in many branches of sciences and provide an important tool for modeling many problems in mathematics, physics, and engineering. These equations appear in fracture mechanics, aerodynamics, the theory of porous filtering, antenna problems in electromagnetic theory, in the quantum effects of electromagnetic fields in the blackbody whose interior is filled by a Kerr nonlinear crystal, in the description of the three-dimensional structure of water around globular solutes, and in the study of a traveling wave solution for a mathematical model describing the population change influenced by a uniformly changing environment (see [4,5,13,25]). Numerical Solution of the three-dimensional integral equations is very significant since they appear in the mathematical modeling. Because these equations are usually difficult to solve practically, the aim of the present research is to develop an accurate method to solve the problem numerically. In this method, we approximate our unknown function with Bernstein's approximation, which will be introduced in the following. One of the advantages of this method is that not only we can get good numerical solutions for second and first kinds of three-dimensional Volterra–Fredholm integral equations, but also we can implement Bernstein's approximation method on these equations with singularity simply and get acceptable solutions for these kinds of equations too.

Several numerical methods for approximating the solution of linear and nonlinear three-dimensional integral equations and specially three-dimensional Volterra–Fredholm integral equations exist in the literature [2,7,8,16,17,20,24]. Also, Bernstein polynomials are studied by many authors and applied to solve different problems; for example, see

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https://doi.org/10.1016/j.amc.2018.07.021
0096-3003/

[1,3,6,10–12,14,15,18,19,21–23]. In the presented paper, we apply Bernstein polynomials to solve 3D-VFIEK1, 3D-VFIEK2 and even singular type of these equations.

The paper is organized as follows: in Section 2, we will introduce the Bernstein's approximation. In Sections 3 and 4, we will perform it on integral equations 2D-VFIEK2 and 3D-VFIEK1, respectively, and demonstrate the solving process by discretization. Also, we find an error bound for proposed method in Section 5. Section 6 offers five examples to show efficiently of approximating the solution of these kinds of integral equations with Bernstein's approximation method.

2. Three-dimensional Bernstein polynomials (3D-BPs)

2.1. Definition and properties

The Bernstein polynomials of degree (m_1, m_2, m_3) are defined on $\Omega = [a_1, b_1] \times [a_2, b_2] \times [a_3, b_3]$ as follows:

$$P_{m_1,m_2,m_3,i_1,i_2,i_3}(x,y,z) = \frac{\binom{m_1}{i_1}\binom{m_2}{i_2}\binom{m_3}{i_3}}{(b_1-a_1)^{m_1}(b_2-a_2)^{m_2}(b_3-a_3)^{m_3}}(x-a_1)^{i_1}(b_1-x)^{m_1-i_1} (y-a_2)^{i_2}(b_2-y)^{m_2-i_2}(z-a_3)^{i_3}(b_3-z)^{m_3-i_3}, \quad (1)$$

where $i_1 = 0, 1, \ldots, m_1,\ i_2 = 0, 1, \ldots, m_2,\ i_3 = 0, 1, \ldots, m_3$ and m_1, m_2, m_3 are arbitrary positive integers. We can write

$$P_{m_1,m_2,m_3,i_1,i_2,i_3}(x,y,z) = P_{m_1,i_1}(x)P_{m_2,i_2}(y)P_{m_3,i_3}(z),$$

where

$$P_{m_1,i_1}(x) = \frac{\binom{m_1}{i_1}}{(b_1-a_1)^{m_1}}(x-a_1)^{i_1}(b_1-x)^{m_1-i_1}, \qquad i_1 = 0, 1, \ldots, m_1,$$
$$P_{m_2,i_2}(y) = \frac{\binom{m_1}{i_1}}{(b_2-a_2)^{m_2}}(y-a_2)^{i_2}(b_2-y)^{m_2-i_2}, \qquad i_2 = 0, 1, \ldots, m_2,$$
$$P_{m_3,i_3}(z) = \frac{\binom{m_3}{i_3}}{(b_3-a_3)^{m_3}}(z-a_3)^{i_3}(b_3-z)^{m_3-i_3}, \qquad i_3 = 0, 1, \ldots, m_3, \quad (2)$$

are one-dimensional Bernstein polynomials with properties

$$\sum_{i_1=0}^{m_1} P_{m_1,i_1}(x) = 1, \quad \sum_{i_2=0}^{m_2} P_{m_2,i_2}(x) = 1, \quad \sum_{i_3=0}^{m_3} P_{m_3,i_3}(x) = 1,$$

respectively.

2.2. Function approximation for 3D-BPs

Let the function of three real variables f be given over Ω, then the Bernstein approximation $B_{m_1,m_2,m_3}(f)$, corresponding to the function f, is defined by means of the formula

$$B_{m_1,m_2,m_3}(f(x,y,z)) = \sum_{i_1=0}^{m_1}\sum_{i_2=0}^{m_2}\sum_{i_3=0}^{m_3} f(x_{i_1}, y_{i_2}, z_{i_3})P_{m_1,m_2,m_3,i_1,i_2,i_3}(x,y,z), \quad (3)$$

where $x_{i_1} = a_1 + i_1\frac{b_1-a_1}{m_1}$, $y_{i_2} = a_2 + i_2\frac{b_2-a_2}{m_2}$, and $z_{i_3} = a_3 + i_3\frac{b_3-a_3}{m_3}$.

3. Solving integral equations 3D-VFIEK2 by 3D-BPs

Let us consider the three-dimensional Volterra–Fredholm integral equations of the second kind (3D-VFIEK2) of the form

$$f(x,y,z) = g(x,y,z) + \lambda_1 \int_{a_1}^{x}\int_{a_2}^{y}\int_{a_3}^{z} k_1(x,y,z,s,t,r)f(s,t,r)drdtds + \lambda_2 \int_{a_1}^{b_1}\int_{a_2}^{b_2}\int_{a_3}^{b_3} k_2(x,y,z,s,t,r)f(s,t,r)drdtds, \quad (4)$$