Applied Mathematics and Computation 338 (2018) 44–54

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

A hybrid binomial inverse hypergeometric probability distribution: Theory and applications

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ARTICLE INFO

Keywords:
Probability distribution function
Hypergeometric distribution
Binomial distribution
Due date

ABSTRACT

In this paper, we define a novel probability distribution triggered by a due date setting problem from a multi-item single level production system. As a result, we derive a recurrence relation for the underlying distribution function and, finally, we also infer the novel discrete distribution function. We underpin our analytical findings by numerical results when computing the distribution function, the expected value as well as the variance. In order to apply and translate our apparatus to other problems, we come up with an application when determining due dates for a production system with sequence-dependent setup costs.

1. Introduction

Probability-driven and combinatorial problems, e.g., defining and analyzing distributions and counting problems of finite sets have been investigated extensively, see [1–3]. For example, combinatorial objects such as permutations, partitions and distributions could be explored enumeratively. Also, one could study distributions as a result of a counting problem and by utilizing combinatorial quantities [1]. Another problem is whether the probabilities can be inferred determinically. In this case, we obtain a distribution based on analytical and closed terms. Otherwise, the probabilities are non-deterministic, so, they need to be inferred statistically [4]. Numerous examples for both cases in many scientific disciplines have been given. We start by mentioning classical discrete and continuous probability distributions such as the Bernoulli- or the hypergeometric distribution [2] and so forth. Another branch thereof which emerged through statistical and information-theoretic applications [4–7] puts the emphasis on quantifying information by using probability distributions. Seminal work in this area was done by Shannon [7] and Brillouin [5] when developing and applying probability distributions to problems in language theory and computer science. Probability distributions have been also used to quantify and characterize the topological information content of relational structures, see, e.g., [8–12]. Finally, we realize that there exists a universe of problems where probability distributions can be applied and described and, hence, they serve as a source for studying problems devoted to translational research.

In this paper, we define a novel probability distribution which partly consists of the binomial and hypergeometric distribution. Yet, the main contribution of the paper relates to describe and characterize the distribution mathematically and apply the underlying mathematical apparatus translationally. So, we prove a recurrence relation and infer a discrete distribution function. Afterwards, we apply the distribution to a due date setting problem from a multi-item single level production

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https://doi.org/10.1016/j.amc.2018.05.063
0096-3003/

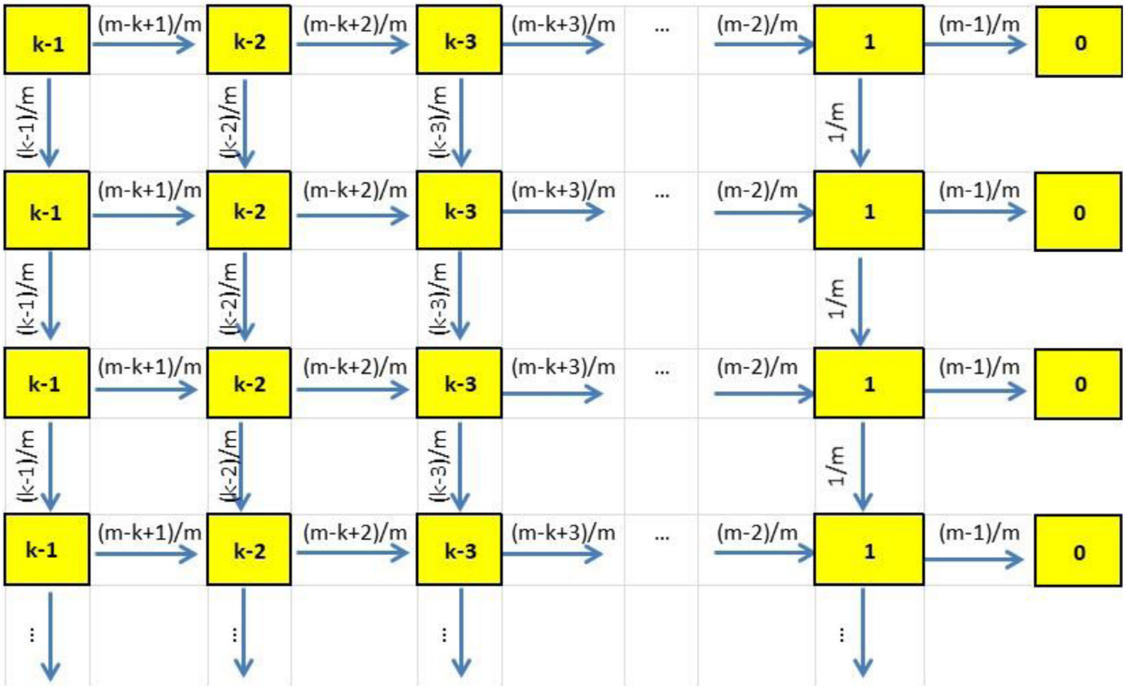

Fig. 1. A probability network.

system [13]. We prove evidence of our analytical findings by generating and discussing numerical results in order to understand the due date problem more deeply.

2. Description of the probability experiment

In order to start describing the probability experiment, we assume $k-1$ black balls and $m-k+1$ white balls in a box. Then a ball is drawn from the box and the subsequent action depends on the color of the selected ball. In case we draw a black ball, it will be returned to the box. So the number of black as well as white balls remains the same. In fact, this can be understood as the binomial part of the probability experiment, i.e., drawing with replacement, see [14]. If a white ball is drawn out of the box, we return the white ball again and one of the black balls will be replaced by a white one. Therefore the number of black balls decreases by one while the number of white balls increases by one. This part of the probability experiment is similar to the experiment which leads to the hypergeometric distribution function, i.e., drawing without replacement, see [14]. Yet, this takes place in an inverse manner because the number of the colored balls which is not selected decreases by one. We repeat this procedure until there is no black ball in the box. So, the random variable we study in this paper equals the number of trials which are needed to reach this result.

This probability experiment can be depicted by Fig. 1. The experiment starts at the left upper corner in the network, where the number of black balls $(k-1)$ is shown in the rectangular box. With the probability of $\frac{(k-1)}{m}$ we select a black ball and the number of black as well as white balls remain the same (illustrated by the box in the second row and first column in the probability network, $k-1$ black balls). The probability that a white ball is chosen in the first trial equals $\frac{m-k+1}{m}$. In this case the number of black balls will be decreased by one and the number of white balls will be increased by one as well (illustrated by the box in the first row and second column in the probability network, $k-2$ black balls). In what follows, we construct the remaining part of the probability network analogously.

When investigating the probability-network, some properties are immediate:

- The number of black balls remains the same for all vertical transitions in the probability network (from one row to another row).
- All vertical transition probabilities of one column are equal.
- The number of black balls decreases by one for each horizontal transition in the probability network (from one column to another column).
- All horizontal transition probabilities from one specific column to the next are the same.

To conclude this section we summarize the defining characteristics of the introduced probability experiment.