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Reduced order Kalman filter for a continuous-time fractional-order system using fractional-order average derivative

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A B S T R A C T

This paper investigates two kinds of reduced order Kalman filters for a continuous-time fractional-order system with uncorrelated and correlated process and measurement noises. The fractional-order average derivative is adopted to enhance the discretization accuracy for the investigated continuous-time fractional-order system. The uncorrelated and correlated cases for the process and measurement noises are treated by the reduced order Kalman filters to achieve the robust estimation for a part of states of a fractional-order system. The truncation issue is considered to implement the practical application of the proposed state estimation algorithm. Finally, two examples for uncorrelated and correlated noises are offered to verify the effectiveness of the proposed reduced order Kalman filters.

1. Introduction

Recently, the research on fractional-order systems has attracted considerable attention because fractional-order systems can better reveal the dynamic characteristics of real-world systems [1]. Compared with integer-order systems, fractional-order systems are more effective to describe the dynamic behaviors of physical systems with memory property [2–4] such as thermal systems [5], electrochemical capacitors [6], lithium-ion batteries [7] and biological population model [8]. The fractional-order derivative is performed with integral operation. In other words, the fractional-order derivative is related not only to the current time, but also to the historical information from the initial time to the current time. Therefore, the fractional-order system can be used to describe physical systems with viscoelasticity and diffusivity [9].

For fractional-order systems, a variety of fractional-order controllers have been explored, and a various types of fractional-order PID controllers are widely used [10,11]. Including fractional-order systems, the information obtained by sensors is always suffered by measurement noises which affect the performance of control systems. Kalman filter is a robust state observer to obtain the state information of control systems with process and measurement noises effectively. By using fractional-order Kalman filters, the accurate state feedbacks can be provided for fractional-order controllers.

In order to obtain the state vector of discrete-time fractional-order systems with process and measurement noises, fractional-order Kalman filters were proposed for linear fractional-order Kalman filters in [12], and the extended fractional-order Kalman filters were also put forward for nonlinear fractional-order systems. In [13], the generalized fractional-order Kalman filters were proposed by using the method of augmented state vector. For the non-Gaussian noises, the modified fractional-order Kalman filters were investigated to deal with Lévy noise and colored noise in [14] and [15] respectively.

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the existence of correlated process and measurement noises was concerned in [16] and [17]. The parameter identification and state estimation of fractional-order systems with correlated noises were both addressed in [18]. Unscented fractional-order Kalman filters were studied to enhance the estimation accuracy of nonlinear fractional-order systems in [19], compared with the extended fractional-order Kalman filters [20]. The Grünwald–Letnikov (G-L) definition is commonly used to discretize continuous-time fractional-order systems [21]. The fractional-order average derivative is used to obtain the discretized fractional-order systems which can improve the accuracy of state estimation.

For the state vector with the high dimension, the high computation complexity of the full order Kalman observer hinders its real application. If a part of the states are required to design a fractional-order controller or to analyze the system performance, it is possible to adopt a reduced order Kalman observer to obtain the effective state estimation. For the robust state estimation issue of integer-order systems, a series of research achievements on reduced order Kalman filters have been reported by Friedland [22], Fairman and Luk [23], and Nagpal et al. [24]. In [25], the unknown input case was investigated for the design of reduced order Kalman filters. The bias estimation was discussed in [26] by reduced order Kalman filters. An alternative novel approach to design the reduced order Kalman filters was proposed in [27], and the reduced order Kalman filter was proposed for non-Guassian systems using Kronecker products in [28]. Reduced order Kalman filters have been widely applied to many practical control systems. In [29], the reduced order extended Kalman filter was used to control the speed of permanent magnet synchronous motor. The state of charge for lithium-ion battery was estimated by the reduced order extended Kalman filter in [30]. In [31], the estimation problem for discrete-time stochastic linear systems with multiple packet dropouts was concerned by reduced order Kalman filters.

The concept of fractional-order average derivative was proposed in [32] to show that the state estimation accuracy of the fractional-order system can be improved by replacing the initial value of the period with the average value within a period. Therefore, the fractional-order average derivative is adopted to discretize continuous-time fractional-order systems with higher estimation accuracy compared with G-L difference [32]. On the basis of the difference equation derived from the fractional-order average derivative, this paper studies the problem on the reduced order Kalman filter and solves the problem of robust state estimation of fractional-order systems under uncorrelated and correlated noises. Two kinds of reduced order Kalman filters are designed with relative simple structures based on the number of observed states, and guarantee the accuracy of state estimation in this paper. Besides, the problem on correlated process and measurement noises is also the main issues of this paper.

The rest of this paper is organized as follows. Section 2 gives the definition of fractional-order derivative and the discretized model of a continuous-time fractional-order system. The reduced order Kalman filters are provided for uncorrelated and correlated process and measurement noises respectively in Section 3. In Section 4, two illustrative examples are offered to verify the effectiveness of the proposed reduced order Kalman filters. Section 5 concludes the whole paper.

2. Problem statement

The Caputo definition is used to describe the dynamics of fractional-order systems with process and measurement noises. For the function $f(t)$, the α-order derivative under Caputo definition is given by [33]

$$ {}_0^C D_t^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \int_0^t (t-\tau)^{n-\alpha-1} f^{(n)}(\tau) \mathrm{d}\tau, \tag{1} $$

where ${}_0^C D_t^{\alpha}$ is α-order Caputo derivative, $\alpha \in \mathbb{R}^+$ is the fractional-order satisfying $n-1 < \alpha \le n$, n is a positive integer, and the Gamma function $\Gamma(\cdot)$ satisfies the condition $\Gamma(r+1) = r\Gamma(r)$ for $r \in \mathbb{R}$, $\mathbb{R}$ is the set of real numbers, $\mathbb{R}^+$ is the set of positive real numbers.

We investigate the state estimation by reduced order Kalman filter for a fractional-order system with process and measurement noises using the Caputo definition. The system and output equations are described by

$$ {}_0^C D_t^{\alpha} x(t) = Ax(t) + Bu(t) + Gw(t), \tag{2} $$

$$ z(t) = C_x x(t) + v(t), \tag{3} $$

where $\alpha \in (0, 2)$ is the fractional-order of system (2), $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^p$ is the control input, $z(t) \in \mathbb{R}^q$ is the measurement output, $w(t) \in \mathbb{R}^m$ and $v(t) \in \mathbb{R}^q$ are the process and measurement noises respectively, $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times p}$, $G \in \mathbb{R}^{n\times m}$, $C_x \in \mathbb{R}^{q\times n}$.

We divide the state vector $x(t)$ as $x(t) = [\bar{x}_1^{\mathrm{T}}(t), \bar{x}_2^{\mathrm{T}}(t)]^{\mathrm{T}}$, where $\bar{x}_1(t) \in \mathbb{R}^{n_1\times n_1}$ is the estimated state affected by $\bar{x}_2(t) \in \mathbb{R}^{n_2\times n_2}$. The corresponding matrices A, B, C_x and G are $A = \begin{bmatrix} A^{11} & A^{12} \\ \mathbf{0} & A^{22} \end{bmatrix}$, $B = \begin{bmatrix} B^1 \\ \mathbf{0} \end{bmatrix}$, $G = \begin{bmatrix} G^1 \\ G^2 \end{bmatrix}$, $C_x = [C, \mathbf{0}]$, where $A^{11} \in \mathbb{R}^{n_1\times n_1}$, $A^{12} \in \mathbb{R}^{n_1\times n_2}$, $A^{22} \in \mathbb{R}^{n_2\times n_2}$, $B^1 \in \mathbb{R}^{n_1\times p}$, $G^1 \in \mathbb{R}^{n_1\times m}$, $G^2 \in \mathbb{R}^{n_2\times m}$, $C \in \mathbb{R}^{q\times n_1}$, $\mathbf{0}$ is the zero matrix with appropriate dimension.

For convenience, the sampling values of $x(t)$, $u(t)$, $z(t)$, $w(t)$ and $v(t)$ are represented by $x(k)$, $u(k)$, $z(k)$, $w(k)$ and $v(k)$ at $t = kT$ for the kth iteration with the sampling period T. The sampling value of the noises $w(k)$ and $v(k)$ are Gaussian white noises, and $E[w(k)] = \mathbf{0}$, $E[v(k)] = \mathbf{0}$, $E[w(k)w^{\mathrm{T}}(l)] = Q\delta(k-l)$ and $E[v(k)v^{\mathrm{T}}(l)] = R\delta(k-l)$, where $\delta(\cdot)$ is the Dirac function with $\delta(k-l) = 1$ for $k = l$ and $\delta(k-l) = 0$ for $k \neq l$.