Applied Mathematics and Computation 339 (2018) 308–322

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Efficient computations for generalized Zernike moments and image recovery

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ARTICLE INFO

Keywords:
Generalized Zernike moments
Generalized Zernike radial polynomials
Dihedral group
Recursive formulae
Jacobi polynomial

ABSTRACT

Zernike moments are a set of orthogonal moments which have been successfully applied in the fields of image processing and pattern recognition. An innovative calculation method for Zernike moments, named generalized Zernike moments, is presented in this study. The generalized Zernike moment is a variant of Zernike moment. In this paper, we are proposing methods to calculate high-order generalized Zernike moments. Two kinds of recurrence for calculating generalized Zernike moments were introduced with rigorous proofs. Through the usage of the symmetries operated by the Dihedral group of order eight, the proposed method is fast and stable. The experimental results show that of the proposed method took 4.206s to compute the top 500-order generalized Zernike moments of an image with 512 by 512 pixels. Furthermore, by choosing the extra parameter α in the recurrence, the method enhanced the accuracy remarkably compared to the regular Zernike moments. Its normalized mean square error is 0.00144067 when α was set to 66 and the top 500-order moments were used to reconstruct the image. This error is 40.47% smaller than the one obtained by using the regular Zernike moments.

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1. Introduction

Zernike polynomials are introduced by the optical physicist Zernike when discussing his phase-contrast method in application to circular concave mirrors [1]. These Zernike polynomials form a basis for the Hilbert space $L^2(D)$ of the square integrable functions over the unit disc. For a function f in $L^2(D)$, the coefficients of the inner product of f with Zernike polynomials are called Zernike moments [2]. Zernike moments are widely applied in many areas of pattern recognition, image processing and computer vision, such as shape recognition [3,4], image retrieval [5,6], trademark image retrieval [7,8] etc. Other variants of Zernike moments, generalized pseudo-Zernike moments introduced in [9], are applied in face recognition. The Zernike polynomials are highly related to the Jacobi polynomials $P_n^{(\alpha,\beta)}(x)$ [1,10,11]; the Kintner's method [11], Prata–Rutsch's method [12] and the parallel recurrence method developed in [13] can backtrack to the recurrence formulae among Jacobi polynomials when computing Zernike moments.

In Wünsche's paper [14] and Janssen's e-print [15], the theoretical considerations of a generalization for the Zernike polynomials, i.e., generalized Zernike polynomials, were developed and discussed to a great extent that is beyond the scope of this study. The generalized Zernike polynomials $R_{nm}^{\alpha}(r)e^{im\theta}$ consist of the radial part and the exponential part, where the radial part with $\alpha > -1$ has more general form than in the regular Zernike polynomials. The generalized Zernike radial polynomials can be regarded as a special case of Jacobi polynomials and are equivalent to the regular Zernike polynomials

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https://doi.org/10.1016/j.amc.2018.07.029
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for the case $\alpha = 0$. In the current literature, there is a lack of using generalized Zernike polynomials to calculate moments that is named generalized Zernike moments. In the calculation of regular Zernike moments, the Kintner's method has been commonly adopted in the field of pattern recognition and demonstrates excellent performance [11,16–18]. To calculate the generalized Zernike moments, a new 3-term recurrence generalizing Kintner's method is proposed in Theorem 1. In [15], the generalized radial polynomial represented by integration formula was given, which generalizes the integration formula for the Zernike radial polynomial in [19]. Base on this result, a 4-term recurrence for the generalized radial polynomial is derived in Theorem 2 which generalizes the formula developed by Shakibaei and Paramesran [20]. With the use of group action symmetry of the order eight Dihedral group [21,22], a computation of generalized Zernike moments can be speeded up by more than eight times. With the extra parameter α in the generalized Zernike polynomials, more theoretical considerations can be undertaken while also providing more choices for applications with better numerical results.

The rest of the paper is organized as follows. Section 2 provides an overview of the basic theory for generalized Zernike moments. In Section 3, we derive two kinds of recurrence formula among the generalized Zernike radial polynomials which play the crucial role of developing the computational theory. In Section 4 two major algorithms are presented. In Section 5, the group action symmetry of Dihedral group is applied to accelerate algorithms. In Section 6, the computation speed of algorithms is compared, and the numerical accuracy is discussed for reconstructed image from its Zernike moments. The choice of parameter α is also addressed to reduce the error for reconstructed images. Section 7 is a comparison summary of the generalized Zernike moments performances.

2. Generalized Zernike polynomials and generalized Zernike moments

Based on the Jacobi polynomial, for the moment order n the repetition m being an integer satisfying $m \equiv n(\mathrm{mod}\,2)$ and $|m| \le n$. Given a real number $\alpha > -1$, the generalized Zernike radial polynomial is given by

$$R^{\alpha}_{nm}(r) = r^{|m|} P^{(\alpha,|m|)}_{\frac{n-|m|}{2}}(2r^2 - 1) \quad for\, 0 \le r < 1. \tag{1}$$

When $\alpha = 0$, $R^{\alpha}_{nm}(r) = R_{nm}(r)$ becomes the regular Zernike radial polynomial. Using the skew-symmetry relation for Jacobi polynomials, one obtains

$$R^{\alpha}_{nm}(r) = (-1)^{\frac{n-|m|}{2}} r^{|m|} P^{(|m|,\alpha)}_{\frac{n-|m|}{2}}(1 - 2r^2) \tag{2}$$

Let the complex number $z = re^{i\theta}$ denote a point within the open unit disc U for $0 \le r < 1$. The generalized Zernike polynomials is defined by

$$V^{\alpha}_{nm}(z) = V^{\alpha}_{nm}(r,\theta) = R^{\alpha}_{nm}(r)e^{im\theta}. \tag{3}$$

Let

$$w(z) = (1 - z\bar{z})^{\alpha} = (1 - r^2)^{\alpha} \tag{4}$$

be defined as the weighted function for $\alpha > -1$ (where $\bar{z}$ denotes the complex conjugation of z). Since Jacobi polynomials $P^{(\alpha,\beta)}_n(x)$ are orthogonal with respect to the weight $(1-x)^{\alpha}(1+x)^{\beta}$ over the closed interval $[-1, 1]$ with $\alpha > 1$ and $\beta > 1$ [10], it yields the orthogonality of the generalized Zernike radial polynomials for the different order and the same repetition

$$\int_0^1 R^{\alpha}_{nm}(r)R^{\alpha}_{n'm}(r)(1 - r^2)^{\alpha} r dr = \frac{(\frac{n+|m|}{2})_{|m|}}{2(n+\alpha+1)(\frac{n+|m|}{2}+\alpha)_{|m|}}\delta_{n'} \tag{5}$$

where $(x)_n$ denotes the falling factorial given in Eq. (6) and $\Gamma(x)$ stands for the Gamma function.

$$(x)_n = \frac{\Gamma(x+1)}{\Gamma(x-n+1)} = x(x-1)(x-2)\cdots(x-n+1) \tag{6}$$

Using the Eq. (5) and the orthogonality of the exponential functions $e^{im\theta}$ on the unit circle, it follows the orthogonal property of generalized Zernike polynomials with respect to the weight $w(z)$:

$$\begin{aligned}\int_0^{2\pi}\int_0^1 V^{\alpha}_{nm}(z)\overline{V^{\alpha}_{n'm'}(z)}(1 - z\bar{z})^{\alpha} r dr d\theta &= \left(\int_0^1 R^{\alpha}_{nm}(r)R^{\alpha}_{n'm'}(r)(1 - r^2)^{\alpha} r dr\right)\left(\int_0^{2\pi} e^{i(m-m')\theta} d\theta\right) \\ &= \left(\frac{(\frac{n+|m|}{2})_{|m|}}{2(n+\alpha+1)(\frac{n+|m|}{2}+\alpha)_{|m|}}\delta_{nn'}\right)(2\pi\delta_{mm'}) \\ &= \frac{\pi(\frac{n+|m|}{2})_{|m|}}{(n+\alpha+1)(\frac{n+|m|}{2}+\alpha)_{|m|}}\delta_{nn'}\delta_{mm'}\end{aligned} \tag{7}$$

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