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A collocation approach for solving two-dimensional second-order linear hyperbolic equations

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ABSTRACT

In this study, a collocation approach is introduced to solve second-order two-dimensional hyperbolic telegraph equation under the initial and boundary conditions. The method is based on the Bessel functions of the first kind, matrix operations and collocation points. The method is constructed in four steps for the considered problem. In first step we construct the fundamental relations for the solution method. By using the collocation points and matrix operations, second step gives the constructing of the main matrix equation. In third step, matrix forms are created for the initial and boundary conditions. We compute the approximate solutions by combining second and third steps. Algorithm of the proposed method is given. Later, error estimation technique is presented and the approximate solutions are improved. Numerical applications are included to demonstrate the validity and applicability of the presented method.

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1. Introduction

In applied sciences, many problems are modeled by the partial differential equations. The computations of solutions of these equations are great importance and therefore many researchers study on the numerical and exact solutions of these equations.

In this study, we consider the second-order two-dimensional hyperbolic telegraph equation

$$\frac{\partial^2 u}{\partial t^2} + 2\alpha \frac{\partial u}{\partial t} + \beta^2 u = \delta \frac{\partial^2 u}{\partial x^2} + \gamma \frac{\partial^2 u}{\partial y^2} + F(x, y, t) \tag{1}$$

with the initial conditions

$$u(x, y, 0) = f_1(x, y), \quad 0 \le x \le 1, \ 0 \le y \le 1, \tag{2}$$

$$\frac{\partial u(x, y, 0)}{\partial t} = f_2(x, y), \ 0 \le x \le 1, \ 0 \le y \le 1, \tag{3}$$

and the boundary conditions

$$u(0, y, t) = g_0(y, t), \ 0 \le y \le 1, \ 0 \le t \le T, \tag{4}$$

$$u(x, 0, t) = h_0(x, t), \ 0 \le x \le 1, \ 0 \le t \le T, \tag{5}$$

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$$u(1,y,t)=g_1(y,t),\ \ 0\le y\le 1,\ \ 0\le t\le T, \tag{6}$$

$$u(x,1,t)=h_1(x,t),\ \ 0\le x\le 1,\ \ 0\le t\le T, \tag{7}$$

where $(x, y, t)\in\Omega=[0, 1]\times[0, 1]\times[0, T]$,

the functions $F(x, y, t)$, $f_1(x,y)$, $f_2(x,y)$, $g_0(y,t)$, $h_0(x,t)$, $g_1(y,t)$ and $h_1(x,t)$ are known functions defined in Ω, the function $u(x, t)$ is unknown function and the constants α, β, δ and γ are known constants.

For $\alpha>0$, $\beta=0$ and $\delta=\gamma=1$, Eq. (1) represents a two-dimensional damped wave equation (with a source term). For $\alpha\ge\beta>0$, $\beta=0$ and $\delta=\gamma=1$, Eq. (1) is named as the two-dimensional hyperbolic telegraph equation.

For partial differential equation problem (1)–(5) in this study, we will obtain the approximate solutions expressed in the truncated Bessel series form

$$u(x,y,t)=\sum_{r=0}^{N}\sum_{p=0}^{N}\sum_{s=0}^{N}a_{r,p,s}J_{r,p,s}(x,y,t);\quad J_{r,p,s}(x,y,t)=J_r(x)J_p(y)J_s(t) \tag{8}$$

where $a_{r,p,\,s}$; $r, p, s=0, ..., N$ are the unknown Bessel coefficients and $J_n(x)$, $n=0, 1, 2, \ldots, N$ are the Bessel functions of first kind defined by

$$J_n(x)=\sum_{k=0}^{[\,|\frac{N-n}{2}|\,]}\frac{(-1)^k}{k!\,(k+n)!}\left(\frac{x}{2}\right)^{2k+n},\ \ n\in N,\ \ 0\le x<\infty.$$

In recent years, the second-order two-dimensional hyperbolic telegraph equations have been solved by using numerical methods such as meshfree techniques [1], meshless collocation approach with barycentric rational interpolation [2], meshless method [3], Euler matrix method [4], fourth-order compact finite difference scheme [5], B-spline differential quadrature method [6], Taylor matrix method [7], fourth-order method based on Hermite interpolation [8], developing Laplace transform [9], differential quadrature method [10], differential invariants [11], Lagrange interpolation and modified cubic B-spline differential quadrature methods [12], local radial basis function collocation method [13], direction implicit methods [14], Laplace transform (LT) inversion technique [15], reduced differential transform method [16], rectangular domain decomposition method [17] and the differential quadrature algorithm [18].

In addition, some ordinary and partial differential equations and integral equations have been solved by using various methods [19–38] by researchers.

2. Solution method

We present the solution method in step by step the following subsections.

2.1. Fundamental matrix relations for solution method

In this part, we construct the matrix relations regarding solution method. Firstly, let us convert the solution function (8) to matrix form as follows:

$$u(x,y,t)=\mathbf{J}(x)\mathbf{Q}(y)\mathbf{S}(t)\mathbf{A} \tag{9}$$

where

$$\mathbf{J}(x)=\begin{bmatrix}J_0(x) & J_1(x) & \cdots & J_N(x)\end{bmatrix}_{1\times(N+1)},\ \ \mathbf{Q}(y)=\begin{bmatrix}\mathbf{J}(y) & 0 & \cdots & 0\\ 0 & \mathbf{J}(y) & \cdots & 0\\ 0 & 0 & \ddots & \vdots\\ 0 & 0 & 0 & \mathbf{J}(y)\end{bmatrix}_{(N+1)\times(N+1)^2},$$
$$\mathbf{S}(t)=\begin{bmatrix}\mathbf{J}(t) & 0 & \cdots & 0\\ 0 & \mathbf{J}(t) & \cdots & 0\\ 0 & 0 & \ddots & \vdots\\ 0 & 0 & 0 & \mathbf{J}(t)\end{bmatrix}_{(N+1)^2\times(N+1)^3} \tag{10}$$
$$\mathbf{A}=\begin{bmatrix}a_{0,0,0} & a_{0,01} & \ldots & a_{0,0,N} & a_{0,1,0} & a_{0,1,1} & \cdots & a_{0,1,N} & \cdots & a_{N,N,0} & a_{N,N,1} & \cdots & a_{N,N,N}\end{bmatrix}^T,$$
$$\mathbf{J}(x)=\mathbf{X}(x)\mathbf{D}^T,\ \ \mathbf{X}(x)=\begin{bmatrix}1 & x & x^2 & \cdots & x^N\end{bmatrix}$$

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