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General Transmission Lemma and Wiener complexity of triangular grids

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ABSTRACT

The Transmission Lemma from Rajasingh et al. (2016) is extended to the General Transmission Lemma. It gives a formula for the transmission of a vertex u as a function of a collection of edge cuts and a u-routing that uniformly intersects the edge cuts. The applicability of the General Transmission Lemma is demonstrated by computing the Wiener complexity of triangular grid networks.

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1. Introduction

The concept of distance pervades mathematics, many fields of science, and even our daily lives. In particular, distances play a vital role in facility location problems (cf. the introduction [7] to the journal's issue dedicated to models, algorithms and applications for location problems), network design in operations research (cf. [13,22]), distance based topological indices in mathematical chemistry (cf. [16,17,32]), measuring closeness of groups of individuals in sociology (cf. [8]), identifying role of players in social networks such as the internet (cf. [6]), and so on.

With Wiener's discovery of a close correlation between the boiling points of certain alkanes and the sum of distances in graphs representing their molecular structure [31], it became apparent that graph invariants (alias topological indices in mathematical chemistry) can be used to predict properties of chemical compounds. Consequently numerous new topological indices have been considered over the past decades and their predictive power for various properties tested, cf. the books [14,15,27]. Many of these invariants are defined via graph distance. In particular, the celebrated Wiener index of a connected graph is defined as the sum of the distances between all unordered pairs of vertices. In the literature one finds general algorithms for computing the Wiener index (see [23]) as well as special algorithms that are faster on specific families of graphs [9,18].

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The transmission $T(u)$ of a vertex $u \in V(G)$ is a concept closely related to the Wiener index but localized to the selected vertex: $T(u)$ is the sum of distances between u and all the other vertices of G, cf. [1,21,26]. In location theory, vertices with the minimum (or maximum) transmission play a special role because they form target sets for locations of facilities. From the Wiener index point of view, the sum of the transmissions of all the vertices of G is twice the Wiener index of G.

Since the transmission is a concept more fundamental than the Wiener index, it deserves a special attention. In our first main result (Lemma 2.1) we generalize the Transmission Lemma [25, Lemma 2.1] and name the new result General Transmission Lemma. This result offers a formula for the transmission of a vertex u in terms of a collection of edge cuts and an u-routing that is compatible with the cuts in a certain way. We then discuss the result and in particular show that the classical cut method can be easily derived from the General Transmission Lemma. Then, in Section 3, we apply the General Transmission Lemma to determine the Wiener complexity of triangular grids. Along the way to obtain this result we also determine the transmission of the vertices of triangular grids.

2. General Transmission Lemma

Let $G = (V(G), E(G))$ be a connected graph. The distance $d_G(u, v)$ (or $d(u, v)$ for short if G is clear from the context) between the vertices u and v of G is the number of edges on a shortest u, v-path. The *diameter diam*(G) of G is the largest distance between the vertices of G. The *Wiener index* $W(G)$ of G is

$$W(G) = \sum_{\{u,v\} \in \binom{V(G)}{2}} d_G(u, v)$$

and the *transmission* $T_G(u)$ (or $T(u)$ for short) of a vertex $u \in V(G)$ is

$$T_G(u) = \sum_{v \in V(G)} d_G(u, v)\,.$$

Suppose that $\{T(u) :\ u \in V(G)\} = \{t_1, \dots, t_k\}$ and that G contains s_i vertices w with $T(w) = t_i$, where $i \in \{1, \dots, k\}$. Then, clearly,

$$W(G) = \frac{1}{2} \sum_{i=1}^{k} s_i t_i\,. \tag{1}$$

To state the General Transmission Lemma, the following concepts are crucial. Let u be a vertex of a connected graph G, and for each $x \in V(G)$, let P_{ux} be a u, x-path. Then the set of paths $\mathcal{P}_u = \{P_{ux} :\ x \in V(G)\}$ is a *u-routing*. A u-routing is *minimal* if each P_{ux} is a u, x-geodesic. Let $\mathcal{P}$ be a u-routing in a (connected) graph G, let $\mathcal{F} = \{F_1, \dots, F_k\}$ be a multi-set of edge cuts of G, and let λ be a positive integer. Then we say that $\mathcal{P}$ is *λ-compatible* with $\mathcal{F}$ if

(i) $|P_{ux} \cap F_i| \le 1$ holds for each i, $1 \le i \le k$, and for each $x \in V(G)$, and
(ii) every edge $e \in \bigcup_{x \in V(G)} E(P_{ux})$ lies in precisely λ edge cuts from $\mathcal{F}$.

With these concepts in hand we are ready for:

Lemma 2.1 (General Transmission Lemma)**.** *Let $\mathcal{F} = \{F_1, \dots, F_k\}$ be a multi-set of edge cuts of a connected graph G. Let $u \in V(G)$ and let G_u^i be the component of $G \backslash F_i$ that contains u. If $\{P_{ux} : x \in V(G)\}$ is a minimal u-routing which is λ-compatible with $\mathcal{F}$, then*

$$T(u) = \frac{1}{\lambda} \sum_{i=1}^{k} \left| V(G) \setminus V(G_u^i) \right|. \tag{2}$$

Proof. By definition of $T(u)$ and by the assumption that $\{P_{ux}\}$ is a minimal u-routing, we have

$$T(u) = \sum_{x \in V(G)} |E(P_{ux})|\,. \tag{3}$$

Let x be an arbitrary vertex of $V(G)$ and consider the path P_{ux}. Let $e \in E(P_{ux})$. Then for every $F_i \in \mathcal{F}$ such that $e \in F_i$, the vertex x lies in the set $V(G) \setminus V(G_u^i)$. Indeed, let $e = yz$, where $d(u, y) < d(u, z)$. Since F_i is an edge cut, y and z lie in different components of $V(G) \setminus V(G_u^i)$. Since $E(P_{ux}) \cap F_i = \{e\}$, the vertices y and u lie in the same component of $V(G) \setminus V(G_u^i)$ which is different from the component in which z and x lie. Consequently, $x \notin V(G_u^i)$.

Since each edge of G lies in λ cuts from $\mathcal{F}$, we infer that x lies in precisely $\lambda \cdot d_G(u, x)$ sets from the family of vertex subsets $\{V(G) \setminus V(G_u^i) :\ x \in V(G), 1 \le i \le k\}$. Consequently,

$$\lambda \cdot \sum_{x \in V(G)} |E(P_{ux})| = \sum_{i=1}^{k} \left| V(G) \setminus V(G_u^i) \right|. \tag{4}$$

Combining (4) with (3) yields the result. □

To see that the condition $|P_{ux} \cap F_i| \le 1$ must be included in the definition of when $\mathcal{P}$ is *λ-compatible* with $\mathcal{F}$ in order that Lemma 2.1 holds true, consider the following example. Let $\mathcal{F} = \{F_1, \dots, F_5\}$ be the collection of edge cuts of the 5-cycle

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