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Finite-time tracking control for stochastic nonlinear systems with full state constraints

Jing Zhang[a], Jianwei Xia[a,*], Wei Sun[a], Guangming Zhuang[a], Zhen Wang[b]

[a] School of Mathematics Science, Liaocheng University, Liaocheng, Shandong 252000, China
[b] College of Information Science and Engineering, Shandong University of Science and Technology, Qingdao 266590, China

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ABSTRACT

In this paper, an adaptive finite-time controller is constructed for stochastic nonlinear systems with parametric uncertainties. All the states in the systems are constrained in a bounded compact set. By constructing a tan-type Barrier Lyapunov Function, the scheme we proposed deals with the finite-time tracking control problem and all the state in the stochastic systems are not violated. Tracking error can converge into a small neighborhood of zero and all the signals in the closed-loop system are bounded. Simulation results demonstrate the effectiveness of the presented approach.

1. Introduction

Many real systems are inevitable to contain the constraints. Violating the constraints on state variables may reduce the performance or cause instability. Driven by these problems, the research on the handling of state constraints is important and has an increasingly attention. For this reason, many method have been proposed to handle the issue of constraints, such as [1–4]. In [1], reference governors-based controller has been proposed by using online optimization algorithms to handle constraints. Recently, the Barrier Lyapunov Functions (BLF) have been proposed to address the problem of constraints, which guarantee that constraints are not violated. For example, in [5], a controller has been constructed for state constrained nonlinear systems in strict-feedback form to achieve output tracking with the help of log-type Barrier Lyapunov Function. Based on the use of Integral-type Barrier Lyapunov Functions, [6] presented a control design for nonlinear systems with state constraints. By using the help of tan-type Barrier Lyapunov Function, [7] has proposed a novel adaptive fault tolerant control scheme for a class of input and state constrained multi-input multi-output nonlinear systems with both multiplicative and additive actuator faults.

It is the fact that we often need error converge in finite time for nonlinear system. As a result, there are a lot of results on the finite-time control have been developed for nonlinear systems [8–12]. In [9], a novel fault tolerant control method has been proposed to deal with the finite-time tracking control problem for a class of joint position constrained robot manipulators with actuator faults. In [12], finite-time tracking controller has been considered for a class of strict-feedback nonlinear systems with full state constraints. Stochastic often occur in practical systems such as manufacturing processes and robot operating systems, which may cause instability and poor performance of the systems [13–16]. Therefore, stochastic are important factors to be taken into consideration, and the investigation on control design for stochastic nonlinear systems has

* Corresponding author.
E-mail addresses: 961736457@qq.com (J. Zhang), njustxjw@126.com (J. Xia), sunwei@lcu.edu.com (W. Sun), zgmtsg@126.com (G. Zhuang), wangzhensd@126.com (Z. Wang).

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received an increasing attention in recent years [17–24]. In [17], an adaptive stabilization controller was constructed for nonlinear high-order systems with stochastic disturbance and uncertain parameters. Liu et al. [18] studied an adaptive neural output feedback tracking controller for uncertain nonlinear multi-input-multi-output stochastic systems with full states constraints. In [22], a new definition of finite-time stability theorem for stochastic nonlinear systems has been proposed, established and proved an important Lyapunov theorem on finite-time stability for stochastic nonlinear systems. However, as we known that the finite-time control tracking problem with state constraints for stochastic nonlinear systems has not been fully investigated, which motivates our current study.

In this paper, the stability of strict-feedback stochastic nonlinear systems with full state constraints is considered. Firstly, the tan-type BLF-based finite-time tracking controller will be designed for nonlinear systems by employing the backstepping technique, which guarantee that the states in stochastic nonlinear systems are not transgressed. Then, we can get that the system output is driven to track a reference signal and all the signals in the closed-loop system are bounded. Finally, the effectiveness of the proposed finite-time control scheme is illustrated via simulation results.

2. Adaptive continuous control

2.1. Problem statement and preliminary results

Consider a class of strict-feedback stochastic nonlinear systems with full state constraints:

$$\begin{cases} dx_i = (f_i(\bar{x}_i) + g_i(\bar{x}_i)x_{i+1})dt + \phi_i^T(\bar{x}_i)d\omega, i = 1, \ldots, n-1, \\ dx_n = (f_n(\bar{x}_n) + g_n(\bar{x}_n)u)dt + \phi_n^T(\bar{x}_n)d\omega, \\ y = x_1, \end{cases} \tag{1}$$

where $x = [x_1, x_2, \ldots, x_n]^T \in R^n$, $u \in R$ and $y \in R$ are state vector, input and output, respectively; $\bar{x}_i = [x_1, \ldots, x_i]^T$; $f_i(\bar{x}_i)$ are the uncertain smooth nonlinear functions and satisfy the following condition:

$$f_i(\bar{x}_i) = \theta^T \varphi_i(\bar{x}_i),$$

where φ_i are known smooth function vector, and θ is an uncertain constant vector satisfying $\theta \in \Omega_\theta = \{\theta \in R^m, \| \theta \| \le \theta_M, \theta_M \in R^+\}$; $g_i(\bar{x}_i)$ are the known smooth nonlinear functions; $\phi_i(\bar{x}_i)$ are known nonlinear function vectors; ω is the standard Wiener process; all the states are constrained in the compact set as $\Omega_x := \{x_i(t) \in R, | x_i(t) | \le k_{c_i}, i = 1, \ldots, n\}$, where k_{c_i} are positive constants.

The control objective of this paper is to design a controller such that output tracking a reference signal within a bounded compact set, all signals in closed-loop system are bounded and all the states are constrained.

To facilitate control system design, the following lemmas and assumption are presented and will be used in the subsequent developments.

Assumption 1. For the continuous function $g_i(\bar{x}_i)$, there exist a positive constant g_0 satisfying $0 < g_0 \le | g_i(\bar{x}_i) |$ for $\bar{x}_i \in \Omega_x$. Without loss of generality, we assume that $g_i(\bar{x}_i)$ are positive.

Assumption 2. The reference signal $y_d(t)$ is continuous and differentiable up to the nth order. There exist positive constants $Y_i, i = 0, \ldots, n$, such that $| y_d(t) | \le Y_0 < k_{c_1}, | y_d^{(i)}(t) | \le Y_i, i = 1, \ldots, n$.

Next, we recall the concept of stochastic nonlinear system. Consider the following stochastic nonlinear system:

$$dx = f(x)dt + g(x)d\omega, \tag{2}$$

where x is the state vector; $f(x) \in R$ and $g(x) \in R^{n \times r}$ satisfy the locally Lipschitz functions and the linear growth condition and satisfy $f(0) = 0, g(0) = 0$; ω is an r-dimensional standard Wiener process.

Definition 1 [25]**.** For any given positive function $V(x, t) \in C^{2,1}$, we define the differential operator L as follows:

$$L[V(x,t)] = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f + \frac{1}{2} Tr\{g^T \frac{\partial^2 V}{\partial x^2} g\},$$

where $Tr(.)$ is the matrix trace.

Lemma 1 *[26]***.** *For any real numbers $x_1, \ldots, x_n$, and $0 < b < 1$, the following inequality holds:*

$$(| x_1 | + \cdots + | x_n |)^b \le | x_1 |^b + \cdots + | x_n |^b .$$

Lemma 2 *[27]***.** *$f(x) \in R$ and $g(x) \in R^{n \times r}$ satisfy the locally Lipschitz functions and the linear growth condition, if there exists a C^2 function V, K_∞ class functions μ_1, μ_2, two constants $c > 0$, $0 < \gamma < 1$, satisfy:*

$$\mu_1(\| x \|) \le V(x) \le \mu_2(\| x \|), \forall x \in R^n,$$
$$LV(x) \le -c(V(x))^\gamma, \forall x \in R^n - \{0\},$$

then, the following properties hold:

(1) The origin of system is finite-time stochastically stable;