Applied Mathematics and Computation 338 (2018) 314–336

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Further analytical bifurcation analysis and applications of coupled logistic maps

A.A. Elsadany[a,b], A.M. Yousef[c], Amr Elsonbaty[d,*]

[a] College of Science and Humanities Studies Al-Kharj, Mathematics Department, Prince Sattam Bin Abdulaziz University, Al-Kharj, Saudi Arabia
[b] Basic Science Department, Faculty of Computers and Informatics, Suez Canal University, Ismailia 41522, Egypt
[c] Faculty of Science, South valley University, Qena, Egypt
[d] Mathematics and Engineering Physics Department, Faculty of Engineering, Mansoura University, Mansoura PO 35516, Egypt

ARTICLE INFO

Keywords:
Neimark–Sacker bifuraction
Marotto's chaos
Image encryption
Coupled logistic map

ABSTRACT

In this work, we extend further the analytical study of complex dynamics exist in two coupled logistic maps. New results about the occurrence of various types of bifurcation in the system, including flip bifurcation, pitchfork bifurcation and Neimark–Sacker bifurcation are presented. To the best of authors' knowledge, the presence of chaotic dynamics in system's behavior has been investigated and proved analytically via Marotto's approach for first time. Numerical simulations are carried out in order to verify theoretical results. Furthermore, chaos based encryption algorithm for images is presented as an application for the coupled logistic maps. Different scenarios of attacks are considered to demonstrate its immunity and effectiveness against the possible attacks. Finally, a circuit realization for the coupled logistic maps is proposed and utilized in a suggested real time text encryption system.

1. Introduction

New emerging nonlinear phenomena can be exhibited by complex systems composed of many nonlinear interacting units where the new properties do not appear in the individual units. There is a wide diversity of such systems that can be found in many fields of science such as biology, physics and chemistry [1–3]. In discrete time dynamical systems, coupling can be included between maps [4–6]. In ecological models, for example, spatial heterogeneity and variety display obvious effects on competing populations [7,8] and coupling process results in increasing the degrees of freedom and obtaining hopeful results. In engineering, several functional mechanism latent phenomena such as the formation of complex patterns, synchronization and coordination have been observed in coupled oscillators and coupled maps [9–20].

Logistic map is one of the well-known simple nonlinear dynamical system that can exhibit chaotic behavior [20–23]. The logistic map was first investigated by May [24] in 1976. It is a first-order difference equation that was primarily proposed to describe population dynamics:

$$x_{n+1} = \mu x_n (1 - x_n),$$

* Corresponding author.
E-mail address: amrelsonbaty@mans.edu.eg (A. Elsonbaty).

https://doi.org/10.1016/j.amc.2018.06.008
0096-3003/

where $0 < x < 1$, represents the ratio of existing population to the maximum possible population. Interesting values for the parameter μ are those in the interval [4]. In logistic map, changing the parameter μ generates very complex dynamics such as a sequence of period-doubling bifurcations, periodic windows and eventual transition to chaos. The simple logistic map can be extended to include additional degrees of freedom [25–28] via considering systems of interacting logistic maps. The system of interacting logistic maps is regarded as a strong point of reference to examine the emerging behaviors arise in other complex systems [23,29]. These coupled maps have wide applications range from noise generators [30,31], encryption machines for secure communications [32–34] to demographic and ecology models [7,23,24,35].

The coupled nonlinear maps can change the dynamics of individual nodes dramatically. For example, symmetric coupling of identical period-doubling systems, replace the conventional period-doubling route to chaos by quasiperiodicity transition [36,37]. For logistic map, there are two forms for coupling two logistic maps, namely, linear coupling and a bilinear coupling. The qualitative and quantitative behaviors of these forms have been examined using numerical [4,38–41] and analytic techniques [42]. In this paper, the focus is on symmetrically coupled nonlinear logistic maps, which can be written in the form

$$\begin{cases} x_{n+1} = a y_n(1 - y_n) + b(x_n - y_n), \\ y_{n+1} = a x_n(1 - x_n) + b(y_n - x_n), \end{cases} \tag{1.1}$$

where $0 < x_n,\ y_n < 1$, $0 \le a \le 4$, and $b \in R$ is the coupling parameter. System (1.1) is symmetric with interchanging of x and y. It is worth noting that although this system has been studied numerically, the vast majority of its possible dynamical behaviors, including pitchfork bifurcation, period doubling bifurcation, Niemark–Sacker bifurcation, multi-basin structures, etc., are not explored analytically yet.

From engineering application's point of view, image encryption based on chaotic maps is very promising tool for cryptography. According to intrinsic futures of images such as enormous data capacity and high redundancy, classical image encryption schemes such as RSA, IDEA, AES and DES are not effective anymore [43,44]. Therefore, numerous schemes of encryption have been modified to overcome the image encryption problems related to traditional methods. In particular, many scientists are interested in chaos based cryptography because of their important properties such as high sensitivity to initial conditions, randomness like behavior, and unpredictability which can satisfy the critical requirements of cryptography such as mixing and diffusion [45–61].

In this paper, the main contributions and results are summarized as follows: It provides the first thoroughly analytical investigation of different types codimension-one bifurcations that are exhibited by the model. It does not rely on numerical simulations, numerical bifurcation tools or the more specialized symmetry techniques to acquire the obtained results. More specifically, by mean of normal form and the center manifold theorems, we prove the occurrence of pitchfork, flip and Neimark–Sacker bifurcations analytically in coupled logistic maps for first time. Also, we presented the first rigorous proof for chaos existence in the coupled maps in the sense of Marotto

The rest of the paper is arranged as follows: in Section 2, the existence and local stability of fixed points are discussed. In Section 3, a detailed bifurcation analysis at the fixed points is investigated. Conditions for presence of Marotto's chaos are derived in Section 4. In Section 5, numerical simulations are carried out so as to verify the analytical results obtained in the article. In Section 6, we present an encryption scheme that relies on the coupled logistic map. The performance of the presented scheme against different possible attacks is also examined. Finally, circuit realization of coupled maps and real-time text encryption application are proposed in Section 7 and the conclusion is given in Section 8.

2. Analytical study of fixed points

In this section, we study the existence of equilibrium points for system (1.1) and their stability properties. By simple calculations, it is found that the system possesses at most four fixed points. This point is clarified as follows:

1. For all values of parameters, the system (1.1) has two fixed points $E_1(0, 0)$ and $E_2(\frac{a-1}{a}, \frac{a-1}{a})$.
2. If $a \ge 1 + 2|b-1|$ or $a \le 1 - -2|b-1|$, then system (1.1) has, additionally, two fixed points

$$E_3\left(\frac{1}{2a}[(a+1-2b) + \sqrt{(a-1)^2 - 4(b-1)^2}], \frac{1}{2a}[(a+1-2b) - \sqrt{(a-1)^2 - 4(b-1)^2}]\right)$$

and

$$E_4\left(\frac{1}{2a}[(a+1-2b) - \sqrt{(a-1)^2 - 4(b-1)^2}], \frac{1}{2a}[(a+1-2b) + \sqrt{(a-1)^2 - 4(b-1)^2}]\right).$$

Investigating the moduli of eigenvalues obtained from characteristic equation at a given equilibrium point represents the basic criteria to study the local stability of this equilibrium point. Recall that Jacobian matrix J of system (1.1) evaluated at a fixed point $E^*(x^*, y^*)$ is given by

$$J(x^*, y^*) = \begin{pmatrix} b & a(1-2y^*) - b \\ a(1-2x^*) - b & b \end{pmatrix}. \tag{2.1}$$

The stability of fixed point $E^*(x^*, y^*)$ is determined according to the following lemma:

Lemma 1 [63]**.** *Let* $F(\lambda) = \lambda^2 + P\lambda + Q$. *Suppose that* $F(1) > 0$, λ_i, $i = 1, 2$ *are the two roots of* $F(\lambda) = 0$. *Then*