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Global Mittag–Leffler stabilization of fractional-order complex-valued memristive neural networks

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ABSTRACT

This paper presents the theoretical results about global Mittag–Leffler stabilization for a class of fractional-order complex-valued memristive neural networks with the designed two types of control rules. As the extension of fractional-order real-valued memristive neural networks, fractional-order complex-valued memristive neural networks have complex-valued states, synaptic weights, and the activation functions. By utilizing the set-valued maps, a generalized fractional derivative inequality as well as fractional-order differential inclusions, several stabilization criteria for global Mittag–Leffler stabilization of fractional-order complex-valued memristive neural networks are established. A numerical example is provided here to illustrate our theoretical results.

1. Introduction

The memristor is a new two-terminal circuit element which was introduced by Chua in 1971 [24]. The new circuit element device is simple and could be described by the relationship between magnetic flux and electric charge in circuit. Its value is related to the polarity and magnitude of the voltage applied to it and the length of time that have been applied by voltage. In addition, the new device has potential application that can provide nonvolatile memory storage and some typical works described a series of features of the memristor could be easily found in [21,51]. Based on these properties, some investigators have introduced memristor in neural network models to form a memristive neural network models(MNNs). Since then, the MNNs have been analyzed in the dynamical behaviours and various literature, see [8–13] and references therein.

As an extension of the integer-order differential and integral calculus, the fractional calculus has received increasing interest in the field of engineering and physics [16–19]. In the past decades, some integer-order calculus systems such as multiagent systems, neural networks and time delayed systems have been studied (see e.g., [40–44]). The superiority of fractional-order calculus in comparison with inventional integer-order calculus is that fractional-order systems have higher accuracy and more degrees of freedom and infinite memory. Moreover, the fractional-order calculus can provide an remarkable instrument in the presentation of memories and hereditary properties. Therefore, taking fractional-order calculus into neural network models could better describe the dynamical behaviour and many excellent results about fractional-order neural networks (FNNs) have been published, such as the synchronization [5–7,49,50], chaos [14], undamped oscillations generated by Hopf bifurcations [35] and Mittag–Leffler stability [2–4,31–33]. In [34], some sufficient conditions have been derived for the boundedness and stability of the fractional-order Hopfield NNs with discontinuous activation func-

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tions by using the Lyapunov methods and a growth condition. Very recently, investigators introduced memristive weights in FNNs to construct the fractional-order memristive neural networks (FMNNs), which have attracted more attentions of many researchers. For example, several results with respect to global Mittag–Leffler stability and synchronization for a class of FMNNs were proposed in [4]. In [1], the authors have investigated the projective synchronization about FMNNs by use of a new hybrid controller and some inequality technique.

On the other hand, the dynamical behaviour analysis of complex-valued neural networks (CVNNs) is one of the most important and active area of research [25–30,36–39]. Compared with the real-valued systems, complex-valued neural networks have different and complicated properties. Particularly in recent years, CVNNs have been analyzed by many researchers in various relevant literature due to its successful applications in imaging, filtering, speech synthesis, optoelectronics, computer version quantum devices and artificial neural information processing. It is well-known that the choice of activation function must be bounded and smooth in real-valued NNs. However, by Liouville's theorem [23], we known that every smooth and bounded activation function reduces to a constant in CVNNs. Therefore, studying the dynamical behaviours of CVNNs are more challenging and necessary. In the analysis of CVNNs, it is essential to ensure the stabilization of the system. On the basis of existing results, there are different methods analyzed the stability of CVNNs. In [20], by using conjugate system, a delay differential inequality and Brouwer's fixed point theorem, some criteria for global exponential stability of the equilibrium for a class of CVNNs with time delays were established. The authors systematically investigated the stability problems of CVNNs by using appropriate Lyapunov function and linear matrix inequality, and provided many useful results in [38]. Furthermore, considering the memristive connection weights, some conditions for the existence, uniqueness, and exponential stability of complex-valued memristive recurrent neural network were concluded based on Lyapunov function and M-matrix in [22,37]. It should be noticed that, the above works mentioned in [20,22,25,29,30,36–39] focus only on the stability of integer-order CVNNs. As we discussed above, many practical problems in control systems are described more accurately by fractional-order calculus than integer-order calculus, hence the dynamical behaviour analysis for fractional-order complex-valued memristive neural networks(CVMNNs) is more valuable. It is a pity that there exist few stability results on the fractional-order CVMNNs, and there are no achievements published on the global Mittag–Leffler stability of fractional-order CVMNNs.

In addition, to the best of our knowledge, a lot of valuable results for nonlinear control strategies have been obtained, for example, state estimation [45,46], impulsive control [47], and switching control [48]. As a simple yet efficient way of stabilization control, the state feedback stabilizing control has been widely applied. However, in many actual applications, for the packed and integrated circuit, it is difficult to find its inner states, then the output of the circuit can be used to measure such an circuit. In this case the output feedback stabilizing control may be necessary. Therefore, some improved stabilization methods for different red systems are also worthy of further research.

Motivated by the above analysis, this paper investigates the problem of stabilization control for a series of fractional-order CVMNNs with the designed two types of control rules. Roughly speaking, the main distribution of this paper can be summarized as follows. Firstly, we construct a fractional-order complex-valued memristive neural network model and define the synaptic weights, the states and the activation red functions in complex domain. Secondly, by utilizing set-valued maps, fractional derivative inequality, fractional-order differential inclusions and appropriate Lyapunov functions, some criteria about global Mittag–Leffler stability of fractional-order CVMNNs are established. Thirdly, we proposed some stabilization techniques which contain state feedback stabilizing control and output feedback stabilizing control.

2. Problem formulation

First of all, we give some definitions of fractional calculus, assumption and lemmas, which are useful to the main results. We give the definition of Euler's gamma function $\Gamma(\alpha)$ as

$$\Gamma(\alpha)=\int_0^{+\infty} h^{\alpha-1}\exp(-h)dh, \qquad (\mathrm{Re}(\alpha)>0),$$

where α is a complex number, and Re(α) represents the real part of α.

Definition 1 [15]**.** The fractional integral $D^{-\alpha}_{t_0,t}$ with fractional order $\alpha\in R^+$ of function $\varrho(t)$ is defined as

$$D^{-\alpha}_{t_0,t}\varrho(t)=\frac{1}{\Gamma(\alpha)}\int_{t_0}^{t}(t-\xi)^{\alpha-1}\varrho(t)d\xi, \quad t\geq t_0.$$

Definition 2 [15]**.** The Caputo derivative of fractional order $\alpha>0$ of function $\varrho(t)$ is defined as

$${}_CD^{\alpha}_{t_0,t}\varrho(t)=\frac{1}{\Gamma(p-\alpha)}\int_{t_0}^{t}(t-\xi)^{p-\alpha-1}\varrho^{(p)}(\xi)d\xi, \quad t\geq t_0,$$

where t_0 is the initial time, $p-1<\alpha<p$, and $p\in Z^+$, $\Gamma(\cdot)$ is Gamma function.

Definition 3 [15]**.** We can define the Mittag–Leffler function as follows:

$$E_\alpha(\vartheta)=\sum_{k=0}^{+\infty}\frac{\vartheta^k}{\Gamma(k\alpha+1)},$$