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On artificial neural networks approach with new cost functions

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ABSTRACT

In this manuscript, the artificial neural networks approach involving generalized sigmoid function as a cost function, and three-layered feed-forward architecture is considered as an iterative scheme for solving linear fractional order ordinary differential equations. The supervised back-propagation type learning algorithm based on the gradient descent method, is able to approximate this a problem on a given arbitrary interval to any desired degree of accuracy. To be more precise, some test problems are also given with the comparison to the simulation and numerical results given by another usual method.

1. Introduction

An overview of recent research is done in the world of applied mathematics that shows that many natural phenomena can be modeled by several types of differential equations. To put it simply, these equations are satisfied tools for studying naturally occurring phenomena in a wide range of applications. Over the years, basic science researchers concluded that differential equations which arise in real world problems rarely have analytic solutions, therefore they must be frequently investigated numerically due to their complexity. In other words, computer simulations and numerical techniques supply a way to solve these problems quickly and easily contrasted to explicit methods. As known, these types of equations are welcomed in very broad field of study. We continue this work with a particular ones, only rarely fractional order problems can be solved analytically, and behavior of closed form solutions may be difficult to understand. To gain greater insight in this area, some considerable research works are listed here. For solving differential equation utilizing ANN combined with Genetic algorithm, Particle swarm optimization, active-set method, sequential quadratic programming, interior-point algorithm and their hybrids [14–22]. Discussed their applications in diversified field of applied science and engineering [26]. Singular Emden–Fowler problem arising in electromagnetic theory was solved [25]. In modeling the fuel ignition which leads the one-dimensional Bratu equation was studied using ANN [24]. Nonlinear differential order system used to model RC circuit and solved [23]. The neural networks approach are a variety of numerical methods known in the field [14–22,27]. As seen, there are many numerical techniques that have been used to solve fractional problems in several cases. Among the above

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survey, we have aimed to extend ANNs approach to the numerical solution of fundamental linear fractional order differential equation with initial condition. This method which has been designed based on the performance of the human brain neuron system, is enable to provide a self-learning iterative technique using empirical evidence. In simple terms, this means that artificial neural networks approach is a non-algorithmic method which uses learning patterns to estimate solution function on a given domain. Recently, several structures of these networks have been used to solve a variety of fractional problems [11,12,29]. The unique feature of these types of networks is that each regular net structure can be considered as an universal approximator [8]. So, a proper model corresponding to the reminded problem can estimate unknown function on a solution domain with very high precision approximate. The goal of this paper is to use a three-layer feed-forward ANN for solving the fractional ordinary differential equation (F.O.D.E) problem. The proposed net has a common structure with one input signal, a finite number of hidden neurons and an output neuron so that the sigmoid activation function is used for hidden layer neurons. This method acts as follow. At first, the origin differential equation is converted to a minimum optimization problem by putting the set of nodal points which have been gotten from discretizing the differential interval. Of course, there are many ways to achieve this goal. Here, will use the standard discretization scheme to partition the given differential domain. However, a suitable technique must be employed to solve the resulting minimization problem for the initial condition. Artificial neural networks approach can provide a powerful tool according to its unique characteristics.The self-learning property of artificial neural networks is the key fact of preferring these in solving many complex mathematical problems. However, we consider the supervised back-propagation type learning algorithm for minimizing the least mean square error which has been used to measure the amount of network error. The learning algorithm which is based on the gradient descent method, has been programmed to be able to reduce this error to zero by adjusting the network parameters. An important thing that should be mentioned here is that, we actually need more than one iteration to approximate the solution. In other words, this numerical method gives us the more accurate answer with increasing number of iterations. Generally, our discussion here emphasizes how a linear fractional order differential equation problem can be solved numerically by a three-layered feed-forward neural architecture.

Our study is organized as follows:

An overview on neural networks and their computational process in solving the mentioned differential equation are stated in Section 2. Two test problems with comparison to the numerical results given by another standard method are presented in Section 3. It has been shown numerically that the proposed ANN architecture presents a desirable convergence of the solution in compared with other method reported in this literature. At last, conclusions regarding the text and recommendations for future works are also pointed out in Section 4.

2. Description of the method

In the recent decade, considerable interest in differential equations of fractional order has been motivated due to their numerous applications in modeling many important phenomena such as diffusion theory, quantitative biology, electro chemistry, scattering theory, probability, potential theory, elasticity, control theory, transport theory, signal processing, kinetic theory, image processing, circuit theory and heat transfer theory. Moreover, these calculus equations also occur greatly in mechanical problems ally with the transmission lines and theory of compressional shock waves. Though some explicit methods have been introduced for these fractional differential equations, but in general there exists no overall method which results in exact solutions for these types of equations. In this section, the artificial neural networks approach which is one of the most efficient self-learning techniques, will be applied to obtain most accurate approximate solution of a linear type differential equation problem in the fractional order case. To illustrate the indicated iterative technique, first basic ideas of this method are summarized in the following.

2.1. Basic idea of ANNs

Indeed, most cases of engineering problems are difficult to be solve analytically. The ANNs approach is one of the well-known ones which has been rapidly growing in recent years. The multi-layer neural architecture is a parallel computational model comprising of linked together layers of neurons. This structure is similar to the nervous system of human brain. The main feature of these networks is their versatile nature where “programming” has been replaced by “learning by patterns”. It should be noted here that any neural network system uses two computational procedures. In the forward process, all input signals are broadcast to output nodes through activated network layers in a forward direction. The error correlation step is backward network parameter (connection weight and bias term) adjustment scheme based on the calculated output error. In a sense, the back-propagation that occurs at each learning cycle, employs a gradient descent rule for reducing the measured network error to a minimum manner. It is to be noted that the above training process is going to be repeated frequently to the point that the desired result is achieved. There are a large number of references dealing with neural net models. For more details see [7,9,30].

To better understand the performance of neural networks, consider the three-layered feed-forward neural architecture shown in Fig. 1. The input signal which is initially produced with a training pattern, is presented to the network and then multiplied by next layer’s connection weight values. Each bias term is added with the corresponding weighted signal and then imported to a suitable activation function to make the outputs of hidden layer. Now, these outputs are weighted by the output layer synapses, and then summed up in the output neuron. According to the above, input-output relations in each