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On a hydrodynamic permeability of a system of coaxial partly porous cylinders with superhydrophobic surfaces

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ABSTRACT

The paper considers a Stokes–Brinkman's system with varying viscosity that describes the continuous flow of viscous incompressible liquid along an ensemble of partially porous cylindrical particles using the cell approach. The analytical solution for the considered system was derived and analyzed for a particular case of Brinkman's viscosity which illustrates the presence of superhydrophobic surfaces in a porous system. Some numerical validation of the derived results are done and the hydrodynamic permeability of the porous system was calculated and analyzed depending on geometrical and physicochemical parameters. Our analysis of the problem shows that the bigger the impermeable core the less the coefficient of hydrodynamic permeability what agrees with the physical process of the filtration. In addition, the bigger the specific permeability of porous layer the greater the hydrodynamic permeability of the porous medium.

1. Introduction

There exist many natural applications where a process of filtration through a porous medium appears. This research topic is a quite attractive among of scientists because of wide class of applications, see e.g. [1–5] and references therein. We shall investigate a flow through a porous medium. The mentioned media is usually modeled as set of particles. The cell model [6] has been very effectively used for studying of mentioned flows. The idea is to replace a system of randomly oriented particles by a periodic array of spheres or cylinders embedded in a centrally-spherical or cylindrical liquid cells. In addition, appropriate boundary conditions on the cell boundary must be stated to take into account the influence of surrounding particles on the flow inside the cell and the force acting on the particle in the center of the cell. There exist four variants of boundary conditions: the Happel (the absence of tangential stresses on the cell surface, [7]), Kuwabara (the absence of vortices, [8]), Cunningham (the flow on the surface of cell is assumed to be uniform, [9,10]) and Kvashnin (the cell symmetry, [11]), see also [5].

Flow through porous shells is frequently modeled by Brinkmans equation, see [12]. However, it has been observed that the results obtained based on the Brinkmans equations do not agree with the experimental data for non-homogeneous porous media. Because of that, a modification of the Brinkmans equation was suggested in [13] for the media having non-homogeneous porosity. One more possible approach to overcome this problem is to use variable viscosity model for the

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Nomenclature

$\sim$	represents the dimensional quantities
r	radial coordinate
z	axial coordinate
φ	angular coordinate
$\mu(r)$	variable viscosity
v^o	velocity of flow in pure liquid layer
v^i	velocity of flow in porous layer
v_z^o, v_z^i	axial components of velocity
v_r^o, v_r^i	radial components of velocity
v_φ^o, v_φ^i	angular components of velocity
p^o	dimensionless pressure of pure liquid
p^i	dimensionless pressure in porous layer
R	dimensionless radius of cylindrical particle
δ	dimensionless radius of porous layer
a, b	dimensionless linear sizes of the cell
k	dimensionless specific permeability of the porous layer
β	dimensionless fitting parameter
ω	pressure gradient
γ	ratio between size of solid core and size of porous layer
α	power index
μ^i	dimensionless variable viscosity of porous layer
μ^o	dimensionless constant viscosity of clear liquid
$\tilde{\sigma}^o$	tangential stresses
$\tilde{U}$	filtration velocity
B_γ	solid core
B_R	porous layer
L_{11}	coefficient of hydrodynamic permeability of the membrane
Q	dimensionless flow rate of fluid passing through the cell
$H^1(B_R)$, $H^1(B_\gamma)$	Sobolev spaces defined in layers B_R, B_γ

liquid/porous boundary region. In case of an arbitrary variable viscosity, the solution to Brinkman–Stokes model can not be solved analytically, except some particular cases.

The flow of aggressive or contaminated liquids through a porous medium leads to a partial dissolution of the surface of the grains (particles or fibers) forming the medium. In addition, the adsorption of flow components on the surface of these grains presents. All this cause a change in the viscosity of the liquid near the surface. The simplest law describing this change in the depth of a cell is a power law function. Earlier such statements of problems were not considered. Usually, the viscosity is assumed to be constant inside the liquid shell surrounding the particle as well as inside the particle itself (see [14]). Brinkman himself studied the case of two different but constant viscosities (see [12]).

The case of flow with variable viscosity was not considered before in details. The recent paper by Filippov and Ryzhikh (see [15]) considers a problem with variable viscosity but for the case of spherical particles. Their model differs from the problem considered in present paper: the viscosity was assumed to be of power low both in porous layer as well as in pure liquid region. The goal of their work was to consider variations in the apparent viscosity along the porous shell of each grain simulated according to Brinkman and to calculate, within the framework of the cell model, variations in the hydrodynamic permeability of a membrane composed of such partially porous spherical particles (grains) as depending on their packing density in the membrane (overall porosity).

In general case, it is not possible to obtain the analytical formula for the solution to Stokes–Brinkman's system with variable viscosity. Therefore, the best possible research of the considered problem is to investigate its qualitative properties: to establish existence and uniqueness of the weak solution, to derive some uniform estimates. The result on existence and uniqueness of the solution of a system describing flow of polynomial and exponential viscosities in a composite cell with partly porous cylindrical particle inside a liquid envelope was proved in [16]. Solvability of analogous problems for flows with arbitrary integrable viscosities was studied in [17]. In particular, the existence and uniqueness for the weak solution was proved for a wide class of viscosity functions. The uniform estimates on growth of the solution depending on the viscosity were also derived in [17]. Analysis in the mentioned literature did not cover any numerical approach. In particular, the behavior of permeability coefficient was not possible to investigate in general case, since the analytical formula can not be derived for an arbitrary viscosity function. The aim of present paper is to obtain the analytical solution for velocity field of Stokes–Brinkman's system in a particular case when viscosity has second-power polynomial growth. This problem was not treated mathematically in the literature before. Besides, the analysis of flow with viscosity as second-order polynomial