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Three-dimensional Green's function approach for analysis of dispersion and attenuation curve in fibre-reinforced heterogeneous viscoelastic layer due to a point source

Santimoy Kundu, Alka Kumari*, Shishir Gupta

Department of Applied Matthematics, IIT(ISM), Dhanbad, Jharkhand 826004, India

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ABSTRACT

The present paper deals with the propagation of Love waves due to the presence of a point source in the fibre-reinforced heterogeneous viscoelastic medium with the aid of Green's function technique. The physical parameters, i.e. rigidity and density are assumed to be exponentially and linearly varying function of depth for medium and half-space, respectively. Three-dimensional Green's function representation for stresses and displacements are derived in complex-plane line-integral. The frequency equations of Love-type waves are derived relating the dependence complex wave numbers after developing the mathematical model with the help of Green's function and Fourier transformation. This representation is useful in various elastodynamic as well as elastostatic problems. The complex expansion of frequency equation is derived to define the phase velocity and attenuation coefficient of Love waves in the proposed model. Dispersion and attenuation curves are plotted by taking different variations in the reinforcement, inhomogeneity and viscoelastic parameters. The results indicate that the effect of these parameters are very pronounced. The final conclusion can be used to understand the nature of propagation of Love waves in the introduced model.

1. Introduction

The study of surface waves is important because of its prospecteous applications in understanding the structure of Earth and physics of earthquakes. But the structure of Earth's deep interior cannot be studied directly. Hence, seismologists use seismic waves to know the infrastructure of the Earth's interior and to study the nature of earthquake sources with ultimate goal of mitigating and eventually controlling the phenomena. Because different types of earthquake waves behave differently when they encounter material in different states (for example, molten, semi-molten, solid), seismic stations established around Earth detect and record the strengths of the different types of waves and the directions from which they came. Geologists use these records to establish the structure of Earth's interior.

Since the Earth's structure is supposed to be made up of different layers which leads to a continuous change in the elastic properties of the material. So in any realistic model of the earth this variation in elastic properties giving rise to inhomogeneity which is principally one-dimentional (i.e. varies with depth). In order to have more profound knowledge about the effect of non-homogeneity on surface waves and profound knowledge about the internal structure of the earth,

* Corresponding author.
E-mail addresses: alkaism@outlook.com, alkapoonam92@gmail.com (A. Kumari).

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it becomes inevitable for the seismologists and geophysicists to consider heterogeneity in the study. Bhattacharya [1] discussed the pronounced effect of heterogeneity on Love wave propagation in intermediate layer placed between two isotropic homogeneous elastic half-spaces. Tapan [2] studied Love waves in heterogeneous media. Many of the researchers focused their concern to show the effect of heterogeinity on propagation of Love waves. Gogna [3], Kausel [4], Chadwick [6], Zhang [5] have presented their precious view on impact of heterogeneity on the propagation of Love waves. The propagation of Love waves in an inhomogeneous layer is important in many practical fields, such as seismology, earthquake engineering, and geotechnical engineering. In addition, the mechanical behaviors of natural geomaterials, such as natural clay deposits formed by sedimentation and sequent consolidation, or rock mass cut by discontinuities, etc., might exhibit both anisotropy and inhomogeneity. In the present paper, the inhomogeneities of the layer and half-space have been taken as a linear and exponential function of depth.

In composite structures we combine two or more materials with significantly different physical or chemical properties that, when combined, produce a material with characteristics different from the individual components. Some of the composite material are cements and concrete, fiber-reinforced composites, self-reinforced composites, metal composites, and ceramic composites. In reinforced structure, its component i.e., concrete and steel are bound together so that they act together as a single anisotropic unit and hence there can be no relative displacement between them as long as they remain in the elastic condition. This characteristic of fiber-reinforced composites leads to high internal damping which help to reduce to the propagation of vibration to neighbouring structure. So, it is great area of interest for the engineers and architects to develope such reinforced elastic materials which resist the transmission of noise and these vibrations [7]. The concepts of developing a continuous self-reinforcement at every point of an elastic solid was given by Belfield et al. [8]. The elastic moduli for fibre-reinforced materials was given by Hasin and Rosen [9]. Chattopadhyay and Choudhury [10] studied the problem of propagation, reflection, and transmission of magneto-elastic shear waves in a self-reinforced medium. The influence of thermally conducting linear fiber-reinforced composite materials on wave propagation was discussed by Singh [11]. Many of the researchers show the impact of fibre-reinforced structure on wave propagation[12–14]. The major reason behind considering the fibre reinforced medium in present article is because of its structural applications in domain of aerospace structural dynamics, automotive engineering, marine engineering, civil engineering and many more engineering fields.

The study of torsional surface waves in viscoelastic media is of great importance because of the involvement of viscoelasticity in the movement and behaviour of the tectonic plates, the plates which float on and travel independently over the mantle of the earth, and which are responsible for earthquakes, volcanoes, etc. The influencing property of absorbing the vibrational energy generated during earthquakes of viscoelastic materials and hence damping the vibrations leads to its structural applications in design process. Few of the viscoelastic materials like copper–manganese alloy, zinc–aluminium alloys etc., are used as dampers in some tall buildings to resist the vibrations. The early works off development of seismic wave propagation in a linearized viscoelastic medium may be found in the works of Cooper [15], Shaw and Bugl [16], Schoenberg [17] and Kaushik and Chopra [18]. The propagation of waves in a homogeneous viscoelastic layer overlying a viscoelastic medium was discussed by Kanai [19]. Propagation of SH-wave in viscoelastic heterogenous layer over half-space with self-weight was investigated by Sahu et al. [20].

The studies regarding the nature of different layers of Earth inspired authors to work on the propagation of Love waves in a fibre-reinforced heterogeneous viscoelastic medium. The schematics of the problem is taken as the propagation of Love waves emitted from an impulsive point source located in an fibre-reinforced heterogeneous viscoelastic half-spaces under heterogeneous viscoelastic layer with different nonhomogeneity, reinforcement and viscoelastic parameters. In this study, the wave number k of the Love wave are taken as complex $k = k_1(1 + i\delta)$. The imaginary part δ defines the attenuation of the Love waves. The problem is solved for medium and half-space with a unit impulse force in space and time followed by Greens function technique. The unit impulse force is represented by Dirac delta function, so an idealized point source or impulse of Love waves can be described by this function. The closed form of velocity equation is obtained which leads to the dispersion equation as its real part and attenuation as its imaginary part.

2. Formulation of the problem

In the present context, characteristics of Love waves propagating in a fibre reinforced heterogeneous viscoelastic layer of thickness H lying over a fibre reinforced heterogeneous viscoelastic half-space with different kinds of heterogeneity are examined. The variations in the rigidities and density for medium are taken to be exponentially varying with depth while for half-space we have taken linear variation with depth including a and ε as heterogeneity parameters for medium and half-space respectively. We have taken an orthogonal Cartesian co-ordinate system $Oxyz$ in such a way that origin O is considered at the common interface of the x, y, z-axis. The interface is at $z = H$ and the rigid surface of the layer is at $z = 0$. The x-axis has been taken along the propagation of waves and z-axis is positive vertically downwards as shown in Fig. 1. The average width of the layer is H and the half-space lies in the region $H \le z \le \infty$. The source of disturbance S is taken at the point of intersection of the interface of separation and x axis.

Now, let u_i, v_i and $w_i (i = 1, 2)$ be the displacements along x, y and z directions, respectively. We consider the possibility of a wave travelling in the direction Ox in such a manner that at any instant, all particles in any line parallel to Oy have equal displacement which implies that all partial derivatives with respect to y vanish. Also, by considering the Love wave condition, we have