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Existence of periodic solutions for a class of second-order p-Laplacian systems[☆]

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ABSTRACT

In this paper, the existence of periodic solutions are obtained for a class of non-autonomous second-order p-Laplacian systems by the least action principle.

1. Introduction

The main purpose of this paper is to consider the existence of periodic solutions for the following problem

$$\begin{cases} \frac{d}{dt}\Big(\Phi_p(\dot{u}(t))\Big) = \nabla F\big(t, u(t)\big), & \text{a.e. } t \in [0, T] \\ u(0) - u(T) = \dot{u}(0) - \dot{u}(T) = 0 \end{cases} \quad (1)$$

where $p > 1$, $\Phi_p(x) = |x|^{p-2}x$, $T > 0$ and $F : [0, T] \times \mathbb{R}^N \to \mathbb{R}$ satisfies the following condition:

(A) $F(t, x)$ is measurable in t for every $x \in \mathbb{R}^N$ and continuous differential in x for a.e. $t \in [0, T]$, and there exist $a \in C(\mathbb{R}^+, \mathbb{R}^+)$, $b \in L^1([0, T]; \mathbb{R}^+)$ such that

$$|F(t, x)| \le a(|x|)b(t), \qquad |\nabla F(t, x)| \le a(|x|)b(t)$$

for all $x \in \mathbb{R}^N$ and a.e. $t \in [0, T]$.

Let φ defined by

$$\varphi(u) = \frac{1}{p}\int_0^T |\dot{u}(t)|^p dt + \int_0^T F\big(t, u(t)\big)dt.$$

It is clear that φ is continuous differentiable and weakly lower semicontinuous on$W_T^{1,p}$ as the sum of a convex continuous function and of a weakly continuous one (see [1]), where

$$W_T^{1,p} = \{u : [0, T] \to \mathbb{R}^N \big| u \text{ is absolutely continuous}, \ u(0) = u(T) \text{ and } \dot{u} \in L^p([0, T]; \mathbb{R}^N)\}$$

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is a Hilbert space and is endowed with the norm

$$\|u\| = \left[\int_0^T |u(t)|^p dt + \int_0^T |\dot{u}(t)|^p dt\right]^{\frac{1}{p}}$$

for each $u \in W_T^{1,p}$. Moreover, we have

$$< \varphi'(u), v >= \int_0^T \left[(|\dot{u}(t)|^{p-2}\dot{u}(t), \dot{v}(t)) + \left(\nabla F\left(t, u(t)\right), v(t)\right)\right]dt$$

for all $u, v \in W_T^{1,p}$. It is well known that solutions of problem (1) correspond to the critical points of φ.

For any $u \in W_T^{1,p}$, let $\bar{u} = \frac{1}{T}\int_0^T u(t)dt$ and $\widetilde{u}(t) = u(t) - \bar{u}$. It is easy to see that

$$\|\widetilde{u}\|_\infty \le T^{\frac{1}{q}}\|\dot{u}\|_{L^p} \qquad \text{(Sobolev's inequality)},$$

$$\|\widetilde{u}\|_{L^p} \le T\|\dot{u}\|_{L^p} \qquad \text{(Wirtinger's inequality)}$$

for all $u \in W_T^{1,p}$ and some positive constant C, where $\frac{1}{p} + \frac{1}{q} = 1$(see [1]).

When $p = 2$, problem (1) becomes the second order Hamiltonian systems. By using the variational methods, the existence and multiplicity of periodic solutions for Hamiltonian systems have been extensively investigated, such as [1,2,3,4,5,8,9] and the references therein. For the general case $p > 1$, there are not so many results. Motivated by results in [6,7,9], we obtain some new existence results for problem (1) by using the least action principle.

2. Main results and proof

First, we recall a definition due to Wu and Tang [7]:

A function $F : \mathbb{R}^N \to \mathbb{R}$ is said to be (λ, μ)-subconvex if

$$F(\lambda(x+y)) \le \mu(F(x) + F(y))$$

for some $\lambda, \mu > 0$ and all $x, y \in \mathbb{R}^N$. A function is said to be γ-subadditive if it is $(1, \gamma)$-subconvex. A function is said to be subadditive if it is 1-subadditive. The convex and subadditive functions are special cases of subconvex functions. However, it is easy to show that the converse is not true. For example, set

$$F(x) = e^{|x|^p} + C_p \ln(1 + |x|^p),$$

where $p > 1$ and C_p is sufficiently large. It follows that F is $(1/2, 1)$-subconvex but neither convex nor γ-subadditive.

Lemma 2.1 *(See [8]). In Sobolev space $W_T^{1,p}$, for $u \in W_T^{1,p}$, $\|u\| \to \infty$ if and only if*

$$\left(|\bar{u}|^p + \int_0^T |\dot{u}(t)|^p dt\right)^{\frac{1}{p}} \to \infty.$$

Theorem 2.2. *Let $F(t, x) = F_1(t, x) + F_2(x)$, where F_1 and F_2 satisfy (A) and the following conditions:*

(i) $F_1(t, \cdot)$ is $(p\lambda, \frac{p\mu}{2})$-subconvex for a.e. $t \in [0, T]$, where $\lambda, \mu > \frac{1}{p}$ and $\mu < p^{p-1}\lambda^p$;

(ii) there exist constants $0 \le r_1 < \frac{1}{T^p}$, $r_2 \in [0, +\infty)$ such that

$$(\nabla F_2(x) - \nabla F_2(y), x - y) \ge -r_1\left|x - y\right|^p - r_2\left|x - y\right|$$

for all $x, y \in \mathbb{R}^N$ and a.e. $t \in [0, T]$;

(iii)

$$\frac{2}{p\mu}\int_0^T F_1(t, p\lambda x)dt + \int_0^T F_2(x)dt \to +\infty$$

as $|x| \to +\infty$.

Then problem (1) has at least one solution which minimizes φ on $W_T^{1,p}$.

Proof. Let $\beta = log_{p\lambda}^{(p\mu)}$, then $0 < \beta < p$. For $|x| > 1$ there exists a positive integer n such that

$$n - 1 < log_{p\lambda}|x| \le n.$$

Then it follows that $|x|^\beta \ge (p\lambda)^{(n-1)\beta} = (p\mu)^{n-1}$ and $|x| \le (p\lambda)^n$. Hence we have

$$F_1(t, x) = F_1\left(t, p\lambda\left(\frac{x}{2p\lambda} + \frac{x}{2p\lambda}\right)\right) \le p\mu F_1\left(t, \frac{x}{2p\lambda}\right) \le \cdots \le (p\mu)^n F_1\left(t, \frac{x}{(2p\lambda)^n}\right) \le p\mu|x|^\beta a_0 b(t)$$