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Global analysis and numerical simulations of a novel stochastic eco-epidemiological model with time delay

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ABSTRACT

In this paper, a new stochastic eco-epidemiological model is proposed and investigated, and the high dimensional model includes time delay and a general incidence rate which make the study more complex. First, the existence and uniqueness of the global positive solution with any positive initial value are verified. Second, using stochastic analysis methods and some differential inequality techniques, we explore the long-time asymptotic properties of the stochastic delayed system. Furthermore, we prove that the solution of the stochastic delayed system has a unique stationary distribution and it has ergodic property. Third, the sufficient conditions of permanence in mean or extinction for the ecological populations are obtained. Finally, we introduce a series of computer simulations to show the performance of the theoretical results.

1. Introduction

Predator–prey is an important interspecific relationship in the ecosystem. In the real world, species for predator–prey system are inevitably affected by various factors. A common factor is the infectious disease, which attracts deep research interests of many scientists. So many authors have done a lot of research in this aspect [1–5]. In this paper, we investigate a three species eco-epidemiological predator–prey system, that is, susceptible prey population, infected prey population and predator population. Moreover, we suppose that the predator only catches the infected prey with the linear mass-action type functional response, and the infected prey does not recover and acquire immunity.

For another, infectious diseases are transmitted to the susceptible prey by some infected individuals after an incubation period. These phenomena can be given by time delay. Recently, authors of [6–10] have directly added the time delay to the models, and the delayed differential equations can describe the reality more vividly. Generally speaking, for the ecological populations, the death should be taken into account during the incubation period. Therefore, the usage of the time delay in the eco-epidemiological model is very meaningful.

As everyone knows the incidence rates play a significant role in the disease transmission process. Numerous authors present that the standard incidence rate or the simple mass action law incidence rate should be replaced by the nonlinear incidence rate [11–14], and through the nonlinear incidence rate which can more accurately reflect the actual situation.

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Therefore in this paper, we consider a general nonlinear incidence rate of the form $\beta x_1(t)f(x_2(t-\omega))$. Now we give an assumption as follows.

Assumption 1.1. Assume that the incidence function $f(x)$ is continuous on $[0,+\infty)$ and it satisfies

(i) $f(0)=0, f'(\zeta)>0, f(\zeta)<\zeta\,(\zeta>0)$.

(ii) $\left(\frac{f(a)}{a}-\frac{f(b)}{b}\right)(f(a)-f(b))\le 0(a,b>0)$.

The function $f(x)$ includes a number of special forms, such as $f(x)=\frac{x}{1+\nu x}(\nu>0)$, $f(x)=x^{\vartheta}(0<\vartheta\le 1)$ and so on.

The differential inequality technique and comparison theorem are widely used to deal with some complex differential systems [15–27]. Recently, the inequality technique was applied to stochastic differential systems [28–31], thus some new results were obtained. In the following, we can use some inequality techniques to deal with the complexity from the nonlinear incidence rate.

Motivated by the above work, a deterministic delayed eco-epidemiological model with a general incidence rate can be described by

$$\begin{cases} \dot{x_1}(t) = a - b_1x_1(t) - \beta e^{-b_1\omega}x_1(t)f(x_2(t-\omega)), \\ \dot{x_2}(t) = \beta e^{-b_1\omega}x_1(t)f(x_2(t-\omega)) - b_2x_2(t) - c_{12}x_2(t)y(t), \\ \dot{y}(t) = c_{21}x_2(t)y(t) - b_3y(t), \end{cases} \tag{1}$$

where $x_1(t)$ and $x_2(t)$ are the population densities of the susceptible prey and the infected prey at time t, respectively, $y(t)$ stands for the population density of the predator at time t. The parameters a, β, $b_i(i=1,2,3)$, c_{12} and c_{21} are all positive constants. Here a is the birth rate of the susceptible prey, β represents the transmission rate of disease, b_1, b_2 and b_3 denote the natural mortality rate of susceptible prey, infected prey and predator, respectively, c_{12} stands for the capturing rate of the predator, $\frac{c_{21}}{c_{12}}$ represents the rate of conversion from infected prey to predator, $\omega\ge 0$ stands for the incubation period of disease. We suppose the infection force of the disease at any time t is described by $\beta e^{-b_1\omega}x_1(t)f(x_2(t-\omega))$, since the population who is exposed at time $t-\omega$ becomes infected prey at time ω latter. The term $0<e^{-b_1\omega}\le 1$ is the the probability of the population who is exposed goes through the incubation period ω that it becomes infected prey which is still alive. The delayed system (1) has three non-negative equilibria, that is, $E_1(\underline{x_1},0,0)$, $E_2(\widehat{x_1},\widehat{x_2},0)$ and $E_3(\overline{x_1},\overline{x_2},\overline{y})$. Moreover, the basic reproduction number is $R_1=\frac{\beta a e^{-b_1\omega}}{b_1b_2}$, which decides the disease occurs or not, and the parameter $R_2=\frac{c_{21}}{b_3}\widehat{x_2}$ decides the positive equilibrium exists or not.

In fact, the ecological populations in the ecosystem are inevitably subjected to uncertain environmental perturbations. It is worth noting that this phenomenon is ubiquitous in the natural environment. Therefore, many stochastic population models have been proposed and studied [32–37]. To the best of our knowledge, the research on global asymptotic behaviors of the high dimensional stochastic eco-epidemiological model with time delay and a general incidence rate is not too much yet. In this paper, we suppose the stochastic perturbations affect the natural mortality rate $b_i(i=1,2,3)$, so that

$$b_1\to b_1+\sigma_1\dot{B_1}(t),\quad b_2\to b_2+\sigma_2\dot{B_2}(t),\quad b_3\to b_3+\sigma_3\dot{B_3}(t),$$

where $B_i(t)(i=1,2,3)$ stands for the standard Wiener processes with $B_i(0)=0$ a.s. $\sigma_i(t)(i=1,2,3)$ denotes a continuous and bounded function for any $t\ge 0$ and $\sigma_i^2(t)(i=1,2,3)$ represents the intensities of Wiener processes. Then corresponding to system (1), a stochastic version can be reached by

$$\begin{cases} \mathrm{d}x_1(t) = \left[a - b_1x_1(t) - \beta e^{-b_1\omega}x_1(t)f(x_2(t-\omega))\right]\mathrm{d}t - \sigma_1x_1(t)\mathrm{d}B_1(t), \\ \mathrm{d}x_2(t) = \left[\beta e^{-b_1\omega}x_1(t)f(x_2(t-\omega)) - b_2x_2(t) - c_{12}x_2(t)y(t)\right]\mathrm{d}t - \sigma_2x_2(t)\mathrm{d}B_2(t), \\ \mathrm{d}y(t) = [c_{21}x_2(t)y(t) - b_3y(t)]\mathrm{d}t - \sigma_3y(t)\mathrm{d}B_3(t). \end{cases} \tag{2}$$

The initial conditions of system (2) are as follows:

$$x_1(\varrho)=\nu_1(\varrho),\quad x_2(\varrho)=\nu_2(\varrho),\quad y(\varrho)=\nu_3(\varrho),\quad \varrho\in[-\omega,0], \tag{3}$$

here $\nu=(\nu_1,\nu_2,\nu_3)^T\in C$ and $\nu_i(\varrho)\ge 0(i=1,2,3)$, C stands for the Banach space $C([-\omega,0],\mathbb{R}_+^3)$ of continuous functions mapping the interval $[-\omega,0]$ into $\mathbb{R}_+^3$ and $\mathbb{R}_+^3=\{\kappa\in\mathbb{R}^3:\kappa\ge 0,\kappa=(x_1,x_2,y)\}$. In view of a biological meaning, now suppose that $\nu_i(0)>0(i=1,2,3)$.

The remainder of this paper is organized as follows. In Section 2, we explore the existence of a unique positive solution of system (2) with initial value (3). In section 3, we investigate the asymptotic behaviors of system (2) around the boundary equilibrium points and the positive equilibrium point of its corresponding deterministic model (1), respectively. Meanwhile, we study the system (2) has a unique stationary distribution and it has ergodic property. The sufficient conditions for the permanence in mean and extinction are proved in Section 4. Finally, numerical simulations are given to illustrate the main conclusions.

Throughout this article, unless otherwise specified, let $(\Omega,\mathscr{F},\{\mathscr{F}\}_{t\ge 0},\mathbb{P})$ be a complete probability space with a filtration $\{\mathscr{F}_t\}_{t\ge 0}$ satisfying the usual conditions (i.e., it is increasing and right continuous while $\mathscr{F}_0$ contains all $\mathbb{P}$-null sets). Further