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Stability and stabilization for discrete-time switched systems with asynchronism☆

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ABSTRACT

In this paper, the $\mathcal{H}_\infty$ control is investigated for a class of discrete-time switched systems. The switching delay between the mode and controller, which leads to the asynchronism, is taken into consideration. The switching signal is considered to be constrained by persistent dwell time (PDT), which is known to be more general than the common used dwell time or average dwell time. Sufficient conditions to guarantee the asymptotic stability and ℓ_2-gain are derived under a PDT scheme. By considering that actual controllers are subjected to norm-bounded gain perturbations, non-fragile controllers are designed based on both state feedback and output feedback stategies. Finally, the effectiveness of the provided methods is illustrated by two examples.

1. Introduction

Switched systems have been attracting considerable attention since they provide a unified framework for mathematical modeling dynamic processes displaying switching features [1–3]. Practical applications of switched systems are extensive, including but not limited to manipulator robots, networked control systems, biological systems, flight control systems and power electronics [4–8]. Quantities of fundamental results about switched systems have been broadly probed [9–13].

The switching mechanisms, which orchestrate the switching among subsystems, play significant roles in stability and system performance of switched systems. Generally, the switching rules could be classified into random and deterministic ones, which result from the system and the designers' intervention [14]. Thereinto, dwell time (DT) and average dwell time switching are two classes of important deterministic switching rules. Primary stability analysis and other issues for switched systems with these two switching rules have been intensively reported. As a matter of fact, there exists another significant switching signal, the persistent dwell time (PDT) switching, which is known to be more general since it can cover both DT and ADT switching as special cases [15,16]. Under the PDT scheme, there exists an infinite number of portions with dwell time no smaller than a fixed value, during which no switching occurs, and the intervals with this property are separated by a period of persistence with upper bound [17]. It indicates that PDT switching could be applied to describe a switched system with both fast and slow switching. Uniformly asymptotically stable criteria are established for the switched systems

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with PDT switching in [15]. In [18], the result is extended to adaptive control with PDT switching for switched nonlinear systems. However, the research on switched systems with this meaningful type of switching signals is still limited so far.

Generally speaking, state feedback controller is widely adopted to guarantee the stability and performance of control system in the existing literature [19,20]. However, it may be difficult or even impossible to obtain the measurement of a fraction of state variables directly under certain circumstances. Therefore, output feedback control is more practical in such situation [21–23]. Meanwhile, it is noted that the controller design by using $\mathcal{H}_\infty$ synthesis techniques may be very sensitive, or fragile, with respect to errors in the controller coefficients [24]. Owing to this, it is necessary to design a controller for a given plant such that the controller is insensitive to some amount of error with respect to its gains, i.e., the designed controller is non-fragile.

Meanwhile, it takes a certain time to identify the active subsystems before applying the corresponding matched controllers, which leads to the switching delay between system modes and controllers, i.e., the asynchronous phenomenon. Considerable efforts have been devoted to address this issue [20,25–29]. To mention a few, in [20], $\mathcal{H}_\infty$ control for discrete-time systems is investigated under asynchronous switching, and in [29], output feedback control is used to stabilize the switched systems with asynchronism. It is particularly noteworthy that aforementioned results are all based on ADT switching. The asynchronous control for switched systems with PDT is more general and meaningful, but to the best of the authors' knowledge, there are no results on this topic.

In this paper, we concern with non-fragile $\mathcal{H}_\infty$ control for discrete-time switching systems with asynchronous switching. Under the PDT switching scheme, a non-weighted disturbance attenuation is derived, which is with more flexible physical meaning than the weighted one widely applied under ADT switching. Mode-dependent controllers are designed based on both state feedback and output feedback. The effectiveness is illustrated by two examples. The remainder of this paper is organized as follows. In Section 2, necessary definitions and lemmas are reviewed for further proceeding. In Section 3, criteria to guarantee the stability and ℓ_2-gain are derived for discrete-time switched systems. In Section 4, sufficient conditions for the solvability of the non-fragile $\mathcal{H}_\infty$ controller design are presented based on both state feedback and output feedback, respectively. In Section 5, two examples are addressed to demonstrate the feasibility and effectiveness of the proposed techniques. Finally, we conclude the paper in Section 6.

Notation: The notation used in this paper is fairly standard. $\mathbb{R}^n$ represents the n-dimensional Euclidean space, $\mathbb{Z}^+$ represents the set of nonnegative integers, and $\mathbb{Z}_{\geq s}$ represents the set $\{k \in \mathbb{Z}^+ | k \geq s\}$. The notation $\|\cdot\|$ refers to the Euclidean vector norm. $l_2[0, \infty)$ is the space of square summable infinite sequence, and for $\omega = \{\omega(k)\} \in l_2[0, \infty)$, its norm is given by $\|\omega\|_2 = \sqrt{\sum_{k=0}^{\infty} \omega^\top(k)\omega(k)}$. A function $\beta : [0, \infty) \to [0, \infty)$ is said to be of class $\mathcal{K}_\infty$ if it is continuous, strictly increasing, unbounded and $\beta(0) = 0$. The superscripts "$\top$" and "-1" represent matrix transposition and matrix inverse, respectively. Meanwhile, we use $*$ as an ellipsis for the terms that are introduced by symmetry in symmetric block matrices or long matrix expressions. The notation $X > 0 (\geq 0)$ means that X is real symmetric and positive definite (semi-positive definite).

2. Problem formulation

Consider a class of discrete-time switched linear systems:

$$x(k+1) = A_{\sigma(k)}x(k) + B_{\sigma(k)}u(k) + D_{\sigma(k)}\omega(k), \tag{1}$$

$$y(k) = C_{y\sigma(k)}x(k) + D_{y\sigma(k)}\omega(k), \tag{2}$$

$$z(k) = C_{z\sigma(k)}x(k) + B_{z\sigma(k)}u(k) + D_{z\sigma(k)}\omega(k), \tag{3}$$

where $x(k) \in \mathbb{R}^n$, $u(k) \in \mathbb{R}^m$, $y(k) \in \mathbb{R}^p$, $z(k) \in \mathbb{R}^q$ are the state, control input, measured output, controlled output, respectively, and $\omega(k) \in \mathbb{R}^r$ is the exogenous disturbance which belongs to $\ell_2[0, \infty)$. $\sigma(k) : [0, \infty) \to \mathcal{L} = \{1, 2, \ldots, M\}$ is the switching signal which is a piecewise constant function depending on time k. $A_{\sigma(k)}$, $B_{\sigma(k)}$, $D_{\sigma(k)}$, $C_{y\sigma(k)}$, $D_{y\sigma(k)}$, $C_{z\sigma(k)}$, $B_{z\sigma(k)}$ and $D_{z\sigma(k)}$ are matrix functions with appropriate dimensions of the jumping process $\sigma(k)$.

In this paper, we aim to study $\mathcal{H}_\infty$ control for asynchronously switched systems with PDT switching, and design a set of state or output feedback controllers for switched linear systems (1)–(3). The following definitions and lemmas are introduced for later development.

Definition 1. [15] Consider switching instants k_0, k_1, ..., k_s, ... with $k_0 = 0$. A positive constant τ_D is said to be the persistent dwell time if there exists an infinite number of disjoint intervals of length no smaller than τ_D on which $\sigma(k)$ is constant, and consecutive intervals with this property are separated by no more than T, where T is called the period of persistence.

As a matter of fact, the interval is divided into a number of stages in PDT switching, while each stage consists of the running time of a certain subsystem (termed as τ-portion) and the period of persistence (termed as T-portion). When considering the system is active at the p-th stage, $p \in \mathbb{Z}_{\geq 1}$, the switching occurs at k_{s_p}, k_{s_p+1}, ..., $k_{s_{p+1}}$. It is worth mentioning that k_{s_p+1} stands for the next switching instant after k_{s_p} at the p-th stage, and $k_{s_{p+1}}$ represents the instant switching into $(p+1)$-th stage. In the τ-portion, i.e., $k \in [k_{s_p}, k_{s_p+1})$, a certain subsystem is activated with the running time $\tau_p \geq \tau_D$. In the T-portion, i.e., $k \in [k_{s_p+1}, k_{s_{p+1}})$, the actual running time T_p satisfies