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# New complex projective synchronization strategies for drive-response networks with fractional complex-variable dynamics

Quan Xu[a,b,c], Xiaohui Xu[a,d,*], Shengxian Zhuang[c], Jixue Xiao[b], Chunhua Song[b], Chang Che[b,c]

[a] Key Laboratory of Fluid and Power Machinery, Ministry of Education, Xihua University, Chengdu 610039, China
[b] School of Mechanical Engineering, Xihua University, Chengdu 610039, China
[c] School of Electrical Engineering, Southwest Jiaotong University, Chengdu 610031, China
[d] School of Automobile and Transportation, Xihua University, Chengdu 610039, China

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ABSTRACT

This paper presents a fully decentralized adaptive scheme to solve the open problem of complex projective synchronization (CPS) in drive-response fractional complex-variable networks (DRFCVNs). Based on local mismatch with the desired state and between coupled nodes, several novel fully decentralized fractional adaptive (FDFA) strategies are proposed to adjust both the feedback control strengths and the coupling weights. By employing Hermitian form Lyapunov functionals and other fractional skills, some sufficient criteria are provided for CPS. Numerical simulation examples are finally employed to illustrate the efficiency of the new synchronization strategies.



## 1. Introduction

Over the past two decades, the existing research on complex networks is pervading mathematics, physics, system sciences, control sciences, nonlinear dynamics and so on [1]. For one thing, the interactions among neighboring nodes increase the complexity of collective dynamical behaviors, which stimulates deep insights into the topology and dynamics of complex networks. For another thing, the external signals to one node can affect its neighbors, subsequently causing the whole network to a synchronous or asynchronous state. As a significant cooperative behavior of the complex networks, synchronization keeps on fascinating the scholars owing to its broad applications ranging from smart grid and image encryption to secure communization [2,3]. Up to now, many well-known results on synchronization have been reported with respect to the synchronization manners, the network models, and the control techniques [2–5].

However, the above-mentioned researches on synchronization are based on the complex networks coupled with integer-order and real-variable dynamical systems. With the advancement of science and technology, high request for the network model and control method has been raised in more and more fields. By analogy with classical integer calculus, fractional calculus can offer an outstanding tool to model the real-world dynamical systems, which thus increases the degree of freedom and reduces the robustness requirement of the controller [6]. Although the stability and synchronization analysis of fractional systems [6–15] has gained considerable popularity in control community, only recently has synchronization and

* Corresponding author at: Key Laboratory of Fluid and Power Machinery, Ministry of Education, Xihua University, Chengdu 610039, China.
*E-mail addresses:* quanxnjd@sina.com (Q. Xu), xhxu@163.com (X. Xu), zhuangsx@sina.com (S. Zhuang).

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control of fractional dynamical networks [16–20] been started to attract increasing interests. Moreover, differential dynamical systems where the state variables are complex numbers can be used to describe a lot of practical problems. Recently, much ongoing work is also focused on stability and synchronization of complex-variable dynamical systems [21–24] or dynamical networks [25–28]. Especially, as a new synchronization phenomenon in complex-variable systems, the CPS was first introduced by Wu and Fu [24], which can be seen as an extension of projective synchronization (PS) [29,30]. In other words, in complex space, the response system can be asymptotically synchronized up to the projection of the drive system by the desired complex scaling factor. As the complex scaling factor is arbitrary and more unpredictable than the real scaling factor, the capacity of the transmitted message is doubled and the safety is greatly strengthened. In Ref. [26], the adaptive feedback control and the nodes-based adaptive techniques combined with pinning control strategy were used to achieve the CPS in a kind of complex-variable drive system and response networks. In [27,28], the CPS in drive-response complex-variable networks with stochastic coupling and complex coupling are further investigated.

During the developments in fractional and complex-variable chaotic systems, both of them gradually penetrate in to each other on the border of their interaction, and form a cross [31–35]. As a result, study on synchronization and control in fractional complex-variable chaotic systems or networks is of great theoretical and practical significance. However, there are very limited results to solve this open problem.

In the existing literature, by introducing the adaptive control to the coupling weights, decentralized adaptive algorithms were developped to realize synchronization of complex networks [36–41]. Unlike the control methods without an adaptive strategy or the centralized adaptive strategies [42,43], the coupling weights are adaptively updated on the basis of the local mismatch among coupled nodes.

Generally, the results for traditional complex networks cannot be directly used in fractional cases. Inspired by Ref. [44–46], we introduce a fractional inequality for the Caputo derivative of a Hermitian form [47]. This result combined with other fractional techniques will provide with us an alternative approach to design adaptive controllers in the fractional complex-variable cases. By introducing FDFA strategy on coupling weights, we investigated the complete synchronization of diffusively coupled fractional complex-variable networks. Based on our work [47], Ding et al. [48] studied the synchronization of fractional order complex-variable dynamical networks by designing the adaptive feedback controller.

The main contributions of this study are summarized as follows. First, by extending the drive-response complex-variable networks to the fractional case, the model of DRFCVNs is proposed. Second, some novel FDFA strategies are designed to adjust both the feedback control strengths and coupling weights for the CPS. Third, by utilizing the fractional Lyapunov functional method and some new fractional skills, the Lyapunov-based adaptive control method for synchronization in DRFCVNs has been developed.

The paper is outlined as the following. Some basics of fractional calculus and the model of DRFCVNs are presented in Section 2. In Section 3, the design of the controllers and the synchronization criteria are given. Section 4 gives some numerical simulations to support the theoretical analysis. Finally, Section 5 concludes all of the results.

This paper utilizes the standard mathematical notations. $\|x\| = \sqrt{x^H x}$ represents the norm of a vector $x$. The matrix $A \in \mathbb{C}^{n\times n}$ is Hermitian if $A = A^H$. $\lambda_{\min}(A)(\lambda_{\max}(A))$ denotes the minimum (maximum) eigenvalue of the matrix $A$. Let $x = x^r + jx^i$, where $j = \sqrt{-1}$, $x^r$ and $x^i$ represent its real and imaginary parts, respectively. $P \in \mathbb{C}^{n\times n}$ is a positive (negative) definite Hermitian matrix and if $P \in \mathbb{C}^{n\times n} > 0 (P \in \mathbb{C}^{n\times n} < 0)$.

## 2. Preliminaries

### *2.1. Fractional calculus [6]*

**Definition 2.1.** For $0 < \alpha < 1$, the unified expression for the fractional integral of an integrable function $g(t)$ is

$$_{t_0}I_t^{\alpha} g(t) = \frac{1}{\Gamma(\alpha)} \int_{t_0}^{t} \frac{g(\tau)}{(t-\tau)^{1-\alpha}} d\tau, \tag{1}$$

where $t \geq t_0$, the gamma function $\Gamma(\alpha) = \int_0^{\infty} t^{\alpha-1} \exp(-t) dt$, $\exp(\,\bullet\,)$ is exponential function, and $_{t_0}I_t^{\alpha}$ denotes the fractional integral operator.

**Definition 2.2.** For $0 < \alpha < 1$, the Caputo definition of fractional derivatives is defined by

$$^{C}_{t_0}D_t^{\alpha} g(t) = \frac{1}{\Gamma(1-\alpha)} \int_{t_0}^{t} \frac{1}{(t-\tau)^{\alpha}} \frac{dg(\tau)}{d\tau} d\tau, \tag{2}$$

where $t \geq t_0$. Notice that, unless otherwise stated, we adopt $\alpha \in (0, 1)$ and $t_0 = 0$.

Let us consider the Laplace transform related to the Caputo derivative

$$\mathcal{L}\left\{ ^{C}_{t_0}D_t^{\alpha} g(t) \right\} = s^{\alpha} G(s) - s^{\alpha-1} g(t_0), \tag{3}$$

where $s$ is complex and $\mathcal{L}\{\cdot\}$ represents the Laplace transform operator.

**Property 2.3.**

$$_{t_0}I_t^{\alpha}\, ^{C}_{t_0}D_t^{\alpha} g(t) = g(t) - g(t_0), \ \forall\, t \geq t_0. \tag{4}$$