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An interval algorithm for uncertain dynamic stability analysis

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A R T I C L E I N F O

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A B S T R A C T

Analysis on the stability of dynamic systems is a very important field in structural elasticity theory. For possible engineering situations where the system parameters are uncertain–but–bounded, the propagation from the initial uncertainties of parameters to the consequent uncertainty of dynamic stability of structure is elaborately investigated by interval analysis for the first time. The interval dynamic stability issue is studied through the column structure as a generalized interval eigenvalue problem, and an analytic theorem is rigorously derived and numerically verified for solving the generalized interval eigenvalue problem. Moreover, to avoid the exaggeration on actual bounds of solution, an effective approach is established to limit the dependency phenomenon in the interval dynamic stability analysis. The interval relationships of the excitation parameter and the critical load frequency with five types of system parameters are further bridged according to the interval arithmetic to propose a three-stage interval scheme for evaluating the effects of interval system parameters on the dynamic stability of structures. Numerical studies demonstrate that the uncertainty of the constant part of load influences the boundaries of principal instability region much more than that of the variable part of load. Within all structural parameters, the uncertainty of column length has the most effect on the boundaries, which could make the critical frequency of load be magnified about ten times the uncertainty of parameter. In particular, if all the system parameters are interval with an identical uncertainty degree, the uncertainty of parameter would be propagated in the system of dynamic stability and enlarged as high as 20 times the uncertainty of parameter. Consequently, the impacts of parameter uncertainties on the dynamic stability of structures are fairly significant.

1. Introduction

The theory of the dynamic stability of elastic systems, which is related to the theories of vibration and stability, is an important branch of the applied theory of elasticity [1]. After the first work by Beliaev [2] in 1924, the theory of dynamic stability has been intensively explored and many significant monographs were presented in [1,3,4]. A typical phenomenon of dynamic instability in engineering lies in the perfectly straight elastic column subject to a periodic axial excitation. The fundamental literature before 1964 on the development of the theory on dynamic stability of columns has been reviewed in the Bolotin's work [1], including the determination of the principal region of instability for hinged columns [2], dynamic stability of column with arbitrary boundary conditions [5] and the viscously damped column [6], etc. Other important

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developments after 1964 involve the dynamic instability of column under follower loads [7], columns carrying concentrated masses [8,9], composite structures [10,11], structures in magnetic fields [12], and so on.

On the other hand, the existence of uncertainties of system parameters is inevitable in the practical engineering problems [13,14]. When applying the theoretical methods of dynamic stability analysis to real engineering projects, it is very important to know whether the conclusion on structural dynamic stability is greatly affected by the parameter uncertainty and how the uncertainties in load or structural parameters influence the boundaries of regions of dynamic instability. The methods of uncertainty analysis are basically grouped into two categories: the probabilistic approach and the non-probabilistic approach. All methods developed on the theory of probability and statistics belong to the probabilistic approach [15]. In this category, all uncertain system parameters are modelled as random variables or random fields with assigned probability distribution functions. In the field of structural dynamic stability, the probabilistic dynamic stability analyses have been extensively explored in [4,16,17]. Yet, these studies only deal with the random excitations and the structural parameters are considered as deterministic.

The second category of uncertainty analysis includes all uncertainty methods not involving the implementations of statistics [18,19]. Such a framework encompasses the fuzzy approach [20], the interval analysis [21,22], and the convex model with ellipsoidal uncertainties [23,24], etc. Different from the probabilistic analysis, the non-probabilistic approach does not require the probabilistic distribution functions of the uncertain parameters, which provides the prominent flexibility on uncertainty modeling for situations where the probabilistic information on uncertain parameters are insufficient. In real engineering practice, the uncertainties of parameters reflecting structural properties are usually discrete but vary within a range. Under this circumstance, it is not realistic to construct their reliable probabilistic functions, and the interval method could be an appropriate tool for evaluating the effects of parameter uncertainties on the structural performances. Interval methods have been widely adopted in the uncertain analysis of structural static stability and dynamic response [25–28]. However, studies on the uncertain dynamic stability by the non-probabilistic method have basically not been reported hitherto. Therefore, in order to fill in such a theoretical gap, an interval computational method is freshly presented for robustly assessing the uncertainty dynamic stability of elastic structures.

The stability of dynamic systems is usually governed by the differential equation with periodic coefficients, called the Mathieu-Hill type equation [1]. The criteria on discriminating the stability of solution of the Mathieu-Hill equation are generally developed by two methods. One is to directly calculate the characteristic exponents, whose mathematical formulations are commonly very complicated, and the other one is to find the condition under which the periodic solutions exist [29]. Based on the second method, Bolotin [1] developed the theory of boundary frequency equation, which has been widely adopted in the dynamic instability analyses for various types of structures. Moreover, solving the equations of boundary frequency is actually dealing with an eigenvalue problem involving the load parameters and structural arguments [30]. When the load or structural parameters are interval uncertain, the investigation on dynamic stability is equal to studying the interval eigenvalue problem.

Owing to the wide applications of the algebraic eigenvalue problem in physics and engineering, a great deal of work has been done in finding the solutions of the eigenvalue problem with interval parameters. For the standard eigenvalue problem, Deif [31] proposed an algorithm to exactly compute the upper and lower bounds of the eigenvalue spectrum. For the more frequently encountered generalized eigenvalue problem, if the matrices are positive definite or non-negatively decomposable, Qiu et al. [32,33] calculated the exact range of interval eigenvalues according to eigenvalue inclusion principle. An approximate method was also developed by Chen et al. [34] to gain the hull of interval eigenvalues by using the matrix perturbation theory. Besides, several approaches based on iterative search were presented by Leng et al. [35–37]. Sofi et al. [38] also proposed an interval method to bound the uncertain eigenvalues by the improved interval analysis via extra unitary interval. In the interval dynamic stability analysis, the corresponding eigenvalue problem is the generalized interval eigenvalue problem with high order matrices. It has been numerically tested by the authors that the dynamic stability problem basically fails to meet the preconditions of the above exact methods. The unsecured precision and expensive computation for large scale matrices also make the iterative approaches not suitable for the uncertain dynamic stability analysis. Therefore, it is desiderated to set up an effective algorithm to determine the exact hull of eigenvalues of the generalized eigenvalue problem for the interval dynamic stability analysis.

In this study, an exact theorem is rigorously derived for solving the generalized interval eigenvalue problem at first. Then the mapping from interval system parameters to the excitation parameter and critical load frequency is set up according to the interval arithmetic. The constructed eigenvalue theorem and interval mapping are further integrated to propose a three-stage interval scheme for evaluating the effects of interval parameters on structural dynamic stability. Moreover, to ensure the 'true' interval bounds of regions of dynamic instability, an effective way is presented to eliminate the interval dependency successfully.

The remainder of this paper is organized as follows. Section 2 clearly states the interval uncertainty problem in the dynamic stability analysis of elastic system. Section 3 presents an analytical algorithm for solving the generalized interval eigenvalue problem. In Section 4, the interval arithmetic is firstly utilized to construct the mapping between the interval system parameters and the interval-valued functions of excitation parameter and critical load frequency, and then a three-stage procedure is proposed to evaluate the effects of interval parameters on the dynamic stability of elastic system. In Section 5, the influences of various uncertain parameters on the boundaries of principal region of dynamic instability are elaborately explored by numerical studies. Some concluding remarks are finally summarized in Section 6.