Applied Mathematics and Computation 338 (2018) 588–606

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Reaction–diffusion equation based image restoration

Xueqing Zhao[a,b,c], Keke Huang[d,*], Xiaoming Wang[e], Meihong Shi[a,b,c], Xinjuan Zhu[a,b,c], Quanli Gao[a,b,c], Zhaofei Yu[f,g,*]

[a] Shaanxi Key Laboratory of Clothing Intelligence, Xi'an Polytechnic University, Xi'an 710048, China
[b] School of Computer Science, Xi'an Polytechnic University, Xi'an 710048, China
[c] National and Local Joint Engineering Research Center for Advanced Networking & Intelligent Information Service, Xi'an Polytechnic University, Xi'an 710048, China
[d] School of Information Science and Engineering, Central South University, Changsha 410083, China
[e] School of Computer Science, Shaanxi Normal University, Xi'an 710062, China
[f] School of Electronics Engineering and Computer Science, Peking University, Beijing 100871, China
[g] Center for Brain-Inspired Computing Research, Department of Automation, Tsinghua University, Beijing 100084, China

ARTICLE INFO

Keywords:
Image restoration
Deblurring
Denoising
Reaction–diffusion equation
Details preserving

ABSTRACT

We present a novel restoration algorithm based on the reaction–diffusion equation theory, denominated RDER, for restoring images which are corrupted by various blur PSFs and different types of noise (including different levels of impulse noise, Gaussian noise and mixed noise). The focus of this work is to propose an image restoration method based on the reaction diffusion equation and to further extend the traditional diffusion equation. Firstly, the RDER model is constructed by using the restoration ability of the diffusion equation, and the image detail preservation ability of the reaction equation; secondly, based on the difference scheme theory, a discrete RDER model is proposed for image restoration and a RDER algorithm for restoring the image is designed; thirdly, we mathematically analyze the RDER model from the existence, stability and uniqueness of solutions of the RDER model; finally, the proposed RDER algorithm is compared with the current famous state-of-the-art restoration algorithms in image restoring and image details preserving. Theoretical analysis and extensive experimental results show that the RDER is an effective image restoration algorithm for image denoising, image deblurring and image details preserving; in particular, the RDER provides a better performance in terms of the impulse noise and mixed noise.

1. Introduction

Image restoration is an important task in image processing. The goal is to restore the original image from noisy and blurred images [1,2]. The result of image restoration can severely affect many applications, such as robotics, medical imaging, and pattern recognition. As the image is affected by noise during the acquisition process, the image quality is degraded. Usually we use a linear model to describe this process, which is modeled by Shama et al. [3]:

$$I = K * C_0 + N \tag{1}$$

* Corresponding author.
E-mail addresses: huangkeke@csu.edu.cn (K. Huang), yuzf12@pku.edu.cn (Z. Yu).

https://doi.org/10.1016/j.amc.2018.06.054

where I is the corrupted image; C_0 is the original image; K represents a blurring operator; N is the noise (including impulse noise, Gaussian noise and their mixed noise). The denoising and deblurring problem is to reconstruct C_0 from the observation I.

Images are often corrupted by different types of noise during their acquisition, signal amplification and transmission. Gaussian noise and impulse noise are mainly two types of noise in corrupted images, which seriously affect image quality [4,5]. In recent years, many restoration algorithms have been proposed and investigated to improve image quality. For the images are corrupted by the Gaussian noise , two most commonly used image restoration algorithms, namely total-variation regularization [6,7] and the frequency domain based image restoration algorithms [8], such as wavelet, curvelet and ridgelet [9]. The former only considers the fidelity of the input data, and is less depend the types of noise, so the restored image details are smoothed, resulting in a large amount of lost detail information. The latter based on the frequency domain image restoration algorithm performs better in the preservation of details, but they may exhibit pseudo-Gibbs phenomena and introduce artifacts in the restored images [9,10]; for the images corrupted by the impulse noise, median filtering [11], bilateral filtering [12], anisotropic diffusion [13,14] and soft switching-based method [15] are commonly used. The main idea of these algorithms is to remove noise by replacing the gray density value of noisy pixels with the median of these values of the pixels contained within the processing window, and to keep the rest of the pixel values unchanged. However, these algorithms fail to preserve image details during denoising.

In addition to being corrupted by the above mentioned noise, blurring is also an important factor in deteriorating image quality. Blur is mainly caused by motion of the object , defocusing of the lens, camera shake, calibration error of the imaging device, or random fluctuation of the medium [16]. Aiming at the problem of image quality degradation caused by blurring, many deblurred image restoration algorithms have been proposed, mainly based on partial differential equation (PDE) models, By exploiting these characteristics, ones have proposed many restoration algorithms for deblurring images, such as the literature [17]–[22]. As a type of effective image deblurring algorithms, the partial differential equation (PDE) based models, such as total variation (TV) model [17], nonlocal regularization model [18], Geometrical PDEs [19], and the improved traditional deblurring algorithms, like the improved Richardson–Lucy algorithm [20], regularized structured total least squares model [21] and Fourier-wavelet reg-ularized deconvolution (ForWaRD) algorithm [22], all of these algorithms are playing a very important role in image deblurring. However, the drawback of these algorithms is that the image details can not be well preserved when the image is corrupted by noise and blurring. Hence, these algorithms can not restore the actual image signals in many applications.

To address the problem of how to preserve image details while denoising and deblurring, in this paper, based on the reaction–diffusion equation theory, we propose a novel algorithm to restore the images from corrupted by various blur PSFs and different types of noise (including different levels of impulse noise, Gaussian noise and their mixed noise), denominated RDER. Our main contributions are summarized as follows:

(1) We construct a reaction–diffusion equation based model for image restoration (RDER), where the diffusion item of the reaction–diffusion equation can effectively restore the corrupted image; and the reaction item of the reaction–diffusion equation can preserve details of the image well. Therefore , the proposed RDER can effectively restore the corrupted images in the image denoising, deblurring and image details preservation.

(2) Considering the discrete characteristics of the image processing, we analyze the constructed RDER model, based on the difference scheme theory, and propose a discrete form of the RDER.

(3) The RDER model's image restoration ability is theoretically analyzed, and the inferable proofs are given using related mathematical methods, including the existence, stability and uniqueness of of the RDER model solution.

(4) We give RDER simulation results on several test images, including four standard gray level images, one aerial photography, two standard color images and one natural image. Experimental results show that the proposed RDER provides a better performance in terms of the image denoising, deblurring and image details preserving, especially in the case of restoration of the impulse noise and mixed noise.

The rest of this paper is organized as follows: Section 2 discusses the related work; the proposed RDER model and algorithm are introduced in Section 3; to analyze the proposed image restoration capabilities of the proposed RDER model, we give a theoretical analysis and inferred proofs in Section 4; Section 5 presents simulation experimental results and compares them with the most current state-of-the-art restoration algorithms. Finally, the conclusions of this paper are given in Section 6.

2. Related work

So far, the most effective image restoration algorithm for Gaussian noise is a set of the state-of-the-art Block Matching 3-D (BM3D) algorithms, which have been developed for different imaging problems. One example is that the authors proposed an image restoration using sparse 3D transform-domain collaborative filtering (BM3DDEB) in [23]. This algorithm improves the regularization after the discrete Fourier domain, however, it may emerge ringing artifacts; another example is the author of [24] has proposed BM3D image denoising with shape-adaptive principal component analysis (PCA), which uses adaptive-shape neighborhoods, and estimated the second-moment matrix of PCA . The shape neighborhoods, and estimates the second moment matrix of PCA. The shape neighborhood is used as training data, this algorithm is very effective for the image denoising. For denoising, the author has proposed BM3D frames and variational image deblurring (IDD-BM3D) in reference to [25], which uses decoupling method and is effective for denoising and deblurring. To sum up, the above BM3D