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Strong convergence of a tamed theta scheme for NSDDEs with one-sided Lipschitz drift☆

Li Tan[a,b], Chenggui Yuan[c,*]

[a] School of Statistics, Jiangxi University of Finance and Economics, Nanchang, Jiangxi 330013, PR China
[b] Research Center of Applied Statistics, Jiangxi University of Finance and Economics, Nanchang, Jiangxi 330013, PR China
[c] Department of Mathematics Swansea University, Swansea SA2 8PP, UK

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ABSTRACT

This paper is concerned with strong convergence of a tamed theta scheme for neutral stochastic differential delay equations with one-sided Lipschitz drift. Strong convergence rate is revealed under a global one-sided Lipschitz condition, while for a local one-sided Lipschitz condition, the tamed theta scheme is modified to ensure the well-posedness of implicit numerical schemes, then we show the convergence of the numerical solutions.

1. Introduction

Numerical analysis plays an important role in studying stochastic differential equations (SDEs) because most equations can not be solved explicitly. The most commonly used method for approximating SDEs is the explicit Euler–Maruyama (EM) method. There are a lot of literature concerning with the explicit EM scheme for all kinds of SDEs, e.g., Hairer et al. [1], Maruyama [9], Milstein [10], and Kloeden and Platen [6]. Most of the early works on explicit EM scheme were about the SDEs with the globally Lipschitz continuous coefficients, since the explicit EM scheme solutions may not converge in the strong sense to the exact solutions with one-sided Lipschitz continuous and superlinearly growing drift coefficients. Moreover, Hutzenthaler et al. [3] pointed out that the absolute moments of the EM scheme at a finite time could diverge to infinity. In order to cope with these difficulties, Higham et al. [2] studied a split-step backward Euler method for nonlinear SDEs, they showed that the implicit EM scheme converged if the drift coefficient satisfied a one-sided Lipschitz condition and the diffusion coefficient was globally Lipschitz. Hutzenthaler et al. [4] proposed a tamed EM scheme in which the drift term is modified to guarantee the boundness of moments. Later, Sabanis [11,12] studied the strong convergence of the tamed EM scheme and extend the tamed EM scheme to SDEs with superlinearly growing drift and diffusion coefficients, respectively. Zong et al. [16] proposed a semi-tamed Euler scheme to solve the SDEs with the drift coefficient equipped with the Lipschitz continuous part and non-Lipschitz continuous part. Although additional computational effort is needed for implicit analysis, the implicit EM schemes have been showed better than the explicit EM scheme which converges strongly to the exact solution of SDEs under non-globally Lipschitz conditions. The implicit EM methods including the backward EM

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* Corresponding author.
E-mail address: C.Yuan@swansea.ac.uk (C. Yuan).

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scheme, the split-step backward EM scheme and the theta scheme have been extensively studied, for example, Mao and Szpruch [8] studied strong convergence and almost sure stability of the backward EM scheme and the theta scheme to SDEs with non-linear and non-Lipschitzian coefficients, to name a few.

Recently, numerical analysis for neutral stochastic differential delay equations (NSDDEs) has also received a great deal of attention, see e.g., Lan and Yuan [7], Wu and Mao [13], Zhou [15], Zong et al. [17], Zong and Wu [18], and the references therein. However, the existing literature are difficult to deal with one-sided Lipschitz and superlinearly drift. To fill the gap, in this paper, we are going to introduce a tamed theta scheme and discuss the strong convergence of this scheme for NSDDEs in which the drift coefficients are one-sided Lipschitz and superlinearly.

The content of our paper is organized as follows. In Section 2, we consider NSDDEs with global one-sided Lipschitz drift, the tamed theta scheme is introduced and strong convergence is investigated. We reveal that the tamed theta solution converges to the exact solution with order α (see (B1) below)under the global one-sided Lipschitz and the superlinearly growth condition. In Section 3, the global one-sided Lipschitiz drift is replaced by the local one-sided Lipschitz drift, under which we show the convergence of the numerical solutions. In order to guarantee the well-posedness of the implicit tamed scheme, we impose a modified tamed theta scheme with a truncated skill.

2. Global one-sided Lipschitz drift

For a fixed positive integer n, let $(\mathbb{R}^n, \langle\cdot,\cdot\rangle, |\cdot|)$ be an n-dimensional Euclidean space. Denote $\mathbb{R}^n\otimes\mathbb{R}^d$ by the set of all $n\times d$ matrices endowed with Hilbert-Schmidt norm $\|A\| := \sqrt{\text{trace}(A^*A)}$ for every $A\in\mathbb{R}^n\otimes\mathbb{R}^d$, in which A^* is the transpose of A. For a fixed $\tau\in(0,\infty)$, which will be referred to as the delay or memory, let $\mathscr{C}=C([-\tau,0];\mathbb{R}^n)$ be all continuous functions from $[-\tau,0]$ to $\mathbb{R}^n$, equipped with the uniform norm $\|\zeta\|_\infty := \sup_{-\tau\le\theta\le0}|\zeta(\theta)|$ for every $\zeta\in\mathscr{C}$. By a filtered probability space, we mean a quadruple $(\Omega,\mathscr{F},\{\mathscr{F}_t\}_{t\ge0},\mathbb{P})$, where $\mathscr{F}$ is a σ-algebra on the outcome space Ω, $\mathbb{P}$ is a probability measure on the measurable space $(\Omega,\mathscr{F})$, and $\{\mathscr{F}_t\}_{t\ge0}$ is a filtration of sub-σ-algebra of $\mathscr{F}$, where the usual conditions are satisfied, i.e., $(\Omega,\mathscr{F},\mathbb{P})$ is a complete probability space, and $\mathscr{F}_0$ contains all $\mathbb{P}$-null sets of $\mathscr{F}$ and $\mathscr{F}_{t_+} := \bigcap_{s>t}\mathscr{F}_s=\mathscr{F}_t$. Let $\{W(t)\}_{t\ge0}$ be a d-dimensional Brownian motion defined on the filtered probability space $(\Omega,\mathscr{F},\{\mathscr{F}_t\}_{t\ge0},\mathbb{P})$.

In this paper, we consider the following NSDDE

$$d[X(t)-D(X(t-\tau))] = b(X(t),X(t-\tau))dt+\sigma(X(t),X(t-\tau))dW(t),\ t\ge0 \tag{2.1}$$

with initial data

$$X_0=\xi=\{\xi(\theta):-\tau\le\theta\le0\}\in\mathcal{L}^p_{\mathscr{F}_0}([-\tau,0];\mathbb{R}^n),\ p\ge2,$$

that is, ξ is an $\mathscr{F}_0$-measurable $\mathscr{C}$-valued random variable such that $\mathbb{E}\|\xi\|^p_\infty<\infty$ for $p\ge2$. Here, $D:\mathbb{R}^n\to\mathbb{R}^n$, and $b:\mathbb{R}^n\times\mathbb{R}^n\to\mathbb{R}^n$, $\sigma:\mathbb{R}^n\times\mathbb{R}^n\to\mathbb{R}^n\otimes\mathbb{R}^d$ are continuous in x and y. Fix $T>\tau>0$, assume that T and τ are rational numbers, and the step size $\Delta\in(0,1)$ be fraction of T and τ, so that there exist two positive integers M, m such that $\Delta=T/M=\tau/m$. Throughout the paper, we shall denote C by a generic positive constant, whose value may change from line to line. Further, for any $x,y,\bar{x},\bar{y}\in\mathbb{R}^n$, we shall assume that:

(A1) For any $s,t\in[-\tau,0]$ and $q>0$, there exists a positive constant K_1 such that

$$\mathbb{E}\|\xi(s)-\xi(t)\|^q_\infty\le K_1|s-t|^q.$$

(A2) $D(0)=0$, and there exists a positive constant $\kappa\in(0,1/2)$ such that

$$|D(x)-D(\bar{x})|\le\kappa|x-\bar{x}|.$$

(A3) There exists a positive constant K_2 such that

$$\langle x-D(y),b(x,y)\rangle\vee\|\sigma(x,y)\|^2\le K_2(1+|x|^2+|y|^2).$$

(A4) There exist positive constants l, K_3 and K_4 such that for $p\ge2$

$$2\langle x-D(y)-\bar{x}+D(\bar{y}),b(x,y)-b(\bar{x},\bar{y})\rangle+(p-1)\|\sigma(x,y)-\sigma(\bar{x},\bar{y})\|^2\le K_3(|x-\bar{x}|^2+|y-\bar{y}|^2),$$

and

$$|b(x,y)-b(\bar{x},\bar{y})|\le K_4(1+|x|^l+|\bar{x}|^l+|y|^l+|\bar{y}|^l)(|x-\bar{x}|+|y-\bar{y}|).$$

Remark 2.1. Due to the existence of implicitness and the neutral term, scopes of Δ and κ in assumption (A2) are given in order to guarantee rationality.

Remark 2.2. If $b(x, y)$ satisfies (A4), then, for any $x,y\in\mathbb{R}^n$, we have

$$\begin{aligned}|b(x,y)| &\le |b(x,y)-b(0,0)|+|b(0,0)|\le K_4(1+|x|^l+|y|^l)(|x|+|y|)+|b(0,0)|\\ &\le C(1+|x|+|x|^{l+1}+|y|+|y|^{l+1}),\end{aligned}$$