Applied Mathematics and Computation 338 (2018) 669–675

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

On the double Roman domination of graphs☆

Jun Yue[a], Meiqin Wei[b,*], Min Li[a], Guodong Liu[c]

[a] School of Mathematics and Statistics, Shandong Normal University, Jinan, Shandong 250358, PR China
[b] College of Arts and Sciences, Shanghai Maritime University, Shanghai 201306, PR China
[c] China Hebei Lang Lang Technology Development Co., Ltd, Tianjin, PR China

ARTICLE INFO

Keywords:
Double Roman domination
Double Roman domination number
Nordhaus–Gaddum type problem
Cograph
Algorithm

ABSTRACT

A double Roman dominating function of a graph G is a labeling f: $V(G) \rightarrow \{0, 1, 2, 3\}$ such that if $f(v)=0$, then the vertex v must have at least two neighbors labeled 2 under f or one neighbor with $f(w)=3$, and if $f(v)=1$, then v must have at least one neighbor with $f(w) \geq 2$. The double Roman domination number $\gamma_{dR}(G)$ of G is the minimum value of $\Sigma_{v \in V(G)} f(v)$ over such functions. In this paper, we firstly give some bounds of the double Roman domination numbers of graphs with given minimum degree and graphs of diameter 2, and further we get that the double Roman domination numbers of almost all graphs are at most n. Then we obtain sharp upper and lower bounds for $\gamma_{dR}(G)+\gamma_{dR}(\overline{G})$. Moreover, a linear time algorithm for the double Roman domination number of a cograph is given and a characterization of the double Roman cographs is provided. Those results partially answer two open problems posed by Beeler et al. (2016).

© 2018 Elsevier Inc. All rights reserved.

1. Introduction

Throughout this paper, all graphs considered are finite, simple and undirected. Let $G=(V(G), E(G))$ be a graph. For every vertex $v \in V(G)$, the *open neighborhood* of v is $N(v)=\{u \in V(G) : uv \in E(G)\}$ and the *closed neighborhood* of v is $N[v]=N(v) \cup \{v\}$. The *degree* of a vertex $v \in V(G)$ is defined as $d_G(v)=|N(v)|$. The *minimum and maximum degree* of a graph G are denoted by $\delta=\delta(G)$ and $\Delta=\Delta(G)$, respectively. A set $S \subseteq V$ in a graph G is called a *dominating set* if $N[S]=V$. The *domination number* $\gamma(G)$ equals the minimum cardinality of a dominating set in G. Except the classical domination, there are many different kinds of domination problems being studied, such as the total domination(coupon coloring) [11,24], the $[a, b]$-domination [13] and the 2-connected dominating sets [19,20].

Let f: $V(G) \rightarrow \{0, 1, 2\}$ be a function having the property that for every vertex $v \in V$ with $f(v)=0$, there exists a neighbor $u \in N(v)$ with $f(u)=2$. Such a function is called a *Roman dominating function on* G. The *weight* of a Roman dominating function is the sum $f(V)=\sum_{v \in V(G)} f(v)$. The minimum weight of a Roman dominating function on G is called the *Roman domination number* of G and is denoted by $\gamma_R(G)$. A Roman dominating function on G with weight $\gamma_R(G)$ is called a γ_R-*function* of G.

☆ Supported by the National Natural Science Foundation of China (Nos. 11626148 and 11701342) and the Natural Science Foundation of Shandong Province (No. ZR2016AQ01)

* Corresponding author.
E-mail addresses: yuejun06@126.com (J. Yue), weimeiqin8912@163.com (M. Wei), liminemily@sdnu.edu.cn (M. Li), dj201701@163.com (G. Liu).

https://doi.org/10.1016/j.amc.2018.06.033
0096-3003/© 2018 Elsevier Inc. All rights reserved.

The Roman domination was originally defined and discussed by Stewart el at. [23], ReVelle and Rosing [22], and subsequently developed by Cockayne et al. [9]. Since then a hundred papers have been published on various aspects of Roman domination in graphs. For more details, please refer to [1,3,6,10,14].

Recently, Beeler el at. defined a new Roman domination in [6]. A function *f*: $V(G) \rightarrow \{0, 1, 2, 3\}$ is a *double Roman dominating function* on a graph *G* if the following conditions hold, where V_i be the set of vertices labeled *i*.

(i) If $f(v) = 0$, then v must have one neighbor in V_3 or at least two neighbors in V_2;

(ii) If $f(v) = 1$, then v must have at least one neighbor in $V_2 \cup V_3$.

The *double Roman domination number* $\gamma_{dR}(G)$ equals the minimum weight of a double Roman dominating function on *G*, and a double Roman dominating function of *G* with weight $\gamma_{dR}(G)$ is called a γ_{dR}-function of *G*.

Beeler el at. [6] showed that $2\gamma(G) \leq \gamma_{dR}(G) \leq 3\gamma(G)$ and defined a graph *G* to be *double Roman* if $\gamma_{dR}(G) = 3\gamma(G)$. They also studied the relationship between double Roman domination and domination or Roman domination. And further, they presented an upper bound on the double Roman domination number of a connected graph *G* in terms of the order of *G* and characterized the graphs attaining this bound. Moreover, they proposed the following problems.

Problem 1. Characterize the double Roman graphs. In particular, characterize the double Roman trees.

Problem 2. For which classes of graphs, or trees, is $\gamma_{dR}(G) \leq n$?

In [27], Zhang el at. characterized the double Roman trees. However, it is open for other classes of graphs. And recently, Ahangar el at. [2] gave that a graph *G* with minimum degree at least 3 satisfies $\gamma_{dR}(G) \leq n$. But it is still open for graphs with given minimum degree at most 2 and other graph classes. In the same paper, the authors showed that it is NP-complete to determine whether a graph has a double Roman function of weight at most *k*. So it is interesting to consider the linear algorithms for computing the double Roman domination numbers on some graph classes.

The graphs with few P_4s are a special class of graphs, which contain cographs, P_4-sparse and P_4-laden graphs etc. The practical applications (to computational semantics, examination scheduling, clustering analysis and group-based cooperation) of these classes of graphs have certainly motivated the theoretical and algorithmic study. For some NP-hard problems on general graphs, there are polynomial time algorithms on these classes of graphs. The details refer to [11,25,26]. In this paper, we mainly consider the double Roman domination of cographs. The *cograph* is a graph without induced paths on four vertices, which is also called P_4-free graph. It was discovered independently by Jung [17], Lerchs [18], etc., since the 1970s.

In this paper, we focus on the double Roman domination and the double Roman domination numbers of graphs. In Section 2, we firstly give some basic definitions and known results, which will be used in the following sections. The first part of Section 3 is devoted to giving some bounds of the double Roman domination numbers of graphs, such as graphs with given minimum degree and graphs with diameter 2. And then we get that the double Roman domination numbers of almost all graphs are at most *n*, which answers Problem 2. Moreover, the sharp upper and lower bounds for $\gamma_{dR}(G) + \gamma_{dR}(\overline{G})$ are given in the second part of Section 3. Finally, a linear time algorithm for the double Roman domination number of a connected cograph is given in Section 4. And further, we also give a characterization of the double Roman cographs.

2. Preliminaries

In this section, we firstly start with some basic concepts and definitions. And then we list some known results, which will be used in the sequel. For notation and terminology not given here, see [7] and [16].

Given a connected graph *G*. The *distance* between two vertices *u* and *v* in *G*, denoted by $d_G(u, v)$, is the length of a shortest path between *u* and *v* in *G*. The greatest distance between any two vertices in *G* is the diameter of *G*, denoted by *diam*(*G*). We write P_n for the path of order *n*, C_n for the cycle of length *n* and $\overline{K_n}$ for the graph with *n* vertices and no edges.

Now, we list some known results about the double Roman domination numbers, which will be used in the following sections.

Theorem 2.1 [6]**.** *For any double Roman dominating function f′, there exists a double Roman dominating function f of no greater weight than f′ for which no vertex is assigned the value 1.*

Theorem 2.2 [2]**.** *For any graph G of order n with maximum degree* Δ,

$$\gamma_{dR}(G) \geq \frac{2n}{\Delta} + \frac{\Delta - 2}{\Delta}\gamma(G).$$

This bound is sharp for even cycles and paths of order 3k.

We refer the following decision problem as the problem DOUBLE ROM-DOM:

Instance: Graph $G = (V, E)$, positive integer $k \leq |V|$.

Question: Does *G* have a double Roman function of weight at most *k*?

Theorem 2.3 [2]**.** *Problem DOUBLE ROM-DOM is NP-Complete for bipartite graphs and chordal graphs.*

In the following sections, we can assume that the set of vertices labeled 1 is an empty-set in any double Roman domination function of a graph by Theorem 2.1. For the sake of convenience, we refer to “DRD” as “double Roman domination” in the sequel.

Download English Version:

https://daneshyari.com/en/article/8900666

Download Persian Version:

https://daneshyari.com/article/8900666

Daneshyari.com