Applied Mathematics and Computation 339 (2018) 302–307

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Erratum

Erratum to "Numerical solution of linear Fredholm integral equation by using hybrid Taylor and Block-Pulse functions" [Appl. Math. Comput. 149 (2004) 799–806]

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ARTICLE INFO

MSC:
45B05
65R20

Keywords:
Block-Pulse functions
Fredholm integral equation
Operational matrix
Product operation
Taylor polynomials

ABSTRACT

This is an erratum to the published paper "Numerical solution of linear Fredholm integral equation by using hybrid Taylor and Block-Pulse functions" by Maleknejad and Mahmoudi, where there are some scientific errors. After considering these errors we attempt to rectify them by presenting correct approach and formulae. Moreover, by means of some numerical examples, we illustrate the accuracy of solutions after applying the correct formulae.

1. Introduction

In recent years, many kinds of basic functions have been employed in numerical solutions of integral equations. In particular, Maleknejad and Mahmoudi in [1], used the hybrid Taylor polynomials and Block-Pulse functions on the interval [0,1), to estimate the solution of the linear Fredholm integral equations of the second kind.

Recall that on the interval [0,1), the set of Block-Pulse functions $b_n(t),\ n = 1, 2, \dots, N$, are defined as follows:

$$b_n(t) = \begin{cases} 1 & \dfrac{n-1}{N} \le t < \dfrac{n}{N}, \\ 0 & \text{otherwise}. \end{cases} \tag{1}$$

Accordingly, with the Taylor polynomials $T_m(t) = t^m$ on this interval, the Hybrid Taylor Block-Pulse functions are defined as follows:

$$b(n, m, t) = \begin{cases} T_m(Nt - (n-1)) & \dfrac{n-1}{N} \le t < \dfrac{n}{N}, \\ 0 & \text{otherwise}, \end{cases} \tag{2}$$

for $m = 0, 1, \dots, M-1$ and $n = 1, 2, \dots, N$. Here, M and N are the order of Taylor polynomials and Block-Pulse functions, respectively.

DOI of original article: 10.1016/S0096-3003(03)00180-2
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https://doi.org/10.1016/j.amc.2018.07.024
0096-3003/

2. Function approximation

As is standard in Numerical Analysis setting, the authors in [1] have approximated a function $f \in L^2[0, 1)$ by means of Taylor extension as:

$$f(t) \simeq \sum_{n=1}^{N} \sum_{m=0}^{M-1} c(n,m)\, b(n,m,t) = C^T B(t), \tag{3}$$

where, $B(t) = [b(1,0,t), \ldots, b(1, M-1, t), b(2,0,t), \ldots, b(N, M-1, t)]^T$ and $c(n, m)$ is defined as follows:

$$c(n,m) = \frac{1}{N^m m!} \left(\frac{\mathrm{d}^m f(t)}{\mathrm{d}t^m} \right) \Bigg|_{\,t = \dfrac{n-1}{N}} . \tag{4}$$

Here, $c(n, m)$ is actually the coefficient corresponding to the monomial t^m in the Taylor series expansion of $f(t)$, around the left end $\frac{n-1}{N}$ of the nth subinterval $[\frac{n-1}{N}, \frac{n}{N})$.

Afterwards, they also approximated the function $k(t, s) \in L^2([0, 1) \times [0, 1))$ as:

$$k(t,s) = B^T(t)\, K\, B(s), \tag{5}$$

where $K = [k_{ij}]$ for $i, j = 0, 1, \ldots, MN-1$, is an $MN \times MN$ matrix with the entries

$$k_{ij} = \frac{1}{N^{u+v} u!\, v!} \left(\frac{\partial^{i+j} k(t,s)}{\partial t^i \partial s^j} \right) \Bigg|_{\,(t,s) = \left(\dfrac{i}{N}, \dfrac{j}{N}\right)} , \tag{6}$$

when $u = i - \lfloor \frac{i}{N} \rfloor N$ and $v = j - \lfloor \frac{j}{N} \rfloor N$ [1, Sec. 2, Eq. (2.4)]. Here, $\lfloor . \rfloor$ denotes the integer part of the number.

But unfortunately, as we will see, the above formula (6) is quite incorrect. Actually, in order to provide a counterexample, suppose that $N = 3$, $M = 2$ and $k(t, s) = t + s$ for $0 \le t, s < 1$. Therefore,

$$B(t) = \left[\begin{cases} 1 & t \in \left[0, \frac{1}{3}\right), \\ 0 & \text{else.} \end{cases} \begin{cases} 3t & t \in \left[0, \frac{1}{3}\right), \\ 0 & \text{else.} \end{cases} \begin{cases} 1 & t \in \left[\frac{1}{3}, \frac{2}{3}\right), \\ 0 & \text{else.} \end{cases} \begin{cases} 3t-1 & t \in \left[\frac{1}{3}, \frac{2}{3}\right), \\ 0 & \text{else.} \end{cases} \right.$$
$$\left. \begin{cases} 1 & t \in \left[\frac{2}{3}, 1\right), \\ 0 & \text{else.} \end{cases} \begin{cases} 3t-2 & t \in \left[\frac{2}{3}, 1\right), \\ 0 & \text{else.} \end{cases} \right]^T,$$

and

$$K = \begin{pmatrix} 0 & \frac{1}{3} & 0 & 0 & 0 & 0 \\ \frac{1}{3} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

Then, according to (5),

$$B^T(t)\, K\, B(s) = \begin{cases} t+s & 0 \le t, s < \frac{1}{3}, \\ 0 & \text{otherwise.} \end{cases}$$

Now, a glance on the already obtained expression for $B^T(t) K B(s)$ shows that it obviously can not be a suitable approximation for $k(t, s)$. In Section 4, we will present a rectified version of the above formula.

3. Fredholm integral equation of the second kind

The authors in [1] have also employed their (initially incorrect) formulae to solve the following linear Fredholm integral equation:

$$q(t)y(t) = x(t) + \lambda \int_0^1 k(t,s)y(s)ds. \tag{7}$$