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Guaranteed cost consensus for second-order multi-agent systems with heterogeneous inertias[☆]

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ABSTRACT

In this paper, the guaranteed cost consensus problem for second-order multi-agent systems with directed topology is considered, in which each agent has a heterogeneous inertia and control gain. The distributed control protocols with the absolute and relative velocity dampings are proposed, respectively. In each kind of protocol, both the communications with and without the input time delay are also considered. By introducing the auxiliary variables and using Lyapunov stability theory, some sufficient conditions are given to achieve the consensus. Moreover, the upper bounds of the guaranteed cost functions are obtained. Finally, some simulation examples are presented to show the effectiveness of the proposed approaches.

1. Introduction

Distributed coordination of multi-agent systems has been intensively studied in the robotics and control communities. The cooperation of a group of agents leads to advantages such as reliability, flexibility, robustness, and has a lot of potential applications in surveillance systems, satellite formation flying, electric power systems, intelligent transportation systems, etc. As one of the most fundamental research issues in the multi-agent systems, the consensus problem has attracted substantial attention due to its extensive applications in spacecraft formation control, traffic control, cooperative control of mobile autonomous robots, sensor networks and other areas [1–4]. For achieving a global goal, the main idea for consensus problem is to design the distributed controllers on each agent based only on its local neighboring information [5–7].

Recently, a considerable amount of significant results have been reported about the consensus problems of multi-agent systems. A pioneering work on the first-order consensus problem with fixed and switching topologies was proposed in [8]. Furthermore, some more relaxed conditions for reaching consensus with switching topologies were presented in [9]. In order to investigate the consensus problems, a number of effective control strategies have been proposed [10–23], to name a few. For instance, Song et al. [10] considered the leader-following consensus for multi-agent systems with nonlinear dynamics via the pinning control and M-matrix method. Wen et al. [11] discussed the consensus problem for multi-agent systems with intermittent communication, and some criteria for achieving consensus were derived. In [12,13], sampled-date control method was proposed for investigation the consensus problems with fixed topology and time-varying topologies.

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Zhang et al. [14] studied the consensus problem based on observer-based output feedback event-triggered control method. Li et al. [15] investigated the second-order leader-following consensus for multi-agent systems based on event-triggered control. In addition, the adaptive control protocols, which are designed only based on the local information of agents, were proposed in [16–18] to discussion the first-order and second-order consensus problems, respectively. The cooperative output regulation problem of singular heterogeneous multi-agent systems was investigated in [19]. By using adaptive backstepping technology, the consensus problem of high-order multi-agent systems with unknown control directions was considered in [20]. The H_∞ consensus for nonlinear stochastic multi-agent systems with time-delay was investigated in [21]. The couple-group consensus problems were studied in [22,23], respectively. In addition, the optimal consensus for multi-agent systems was investigated in [24,25]. It should be noted that the aforementioned literatures [8–25] only considered the consensus problems for multi-agent systems by using different control schemes.

In designing a controller for a real plant, it is invariably necessary to design a control system which is not only to achieve the control system performance required but also to guarantee the cost of the system. One way to deal with this is so-called guaranteed-cost control approach proposed by Chang and Peng [26]. Hence, the consensus regulation performance and the control energy consumption should be considered in the multi-agent systems. In this case, the consensus problem can be regarded as an optimal problem with multiple objectives. The guaranteed cost control is an effective approach to deal with the problems with multiple objectives. To the best our knowledge, there are some works about the guaranteed cost consensus [27–40]. For example, Wang et al. [27,28] studied the guaranteed cost consensus of first-order multi-agent systems with undirected fixed and switching topologies, in which both the communication with and without time delays were discussed. Xie and Yang [29] considered the guaranteed cost consensus problem for multi-agent systems with actuator faults. In [30,31], the authors investigated the guaranteed cost consensus for the multi-agent systems with and without time-delays via event-triggered control. In [32,33], the guaranteed cost consensus for continuous time and discrete-time high-dimensional multi-agent systems with time-varying delays were studied, respectively. The sampled data control was proposed for investigating the guaranteed cost consensus in [34]. Xi et al. [35] studied the guaranteed cost consensus for singular multi-agent system, and some sufficient conditions were obtained. In [36–38], the leader-following guaranteed cost consensus were considered for linear and nonlinear multi-agent systems. In addition, the second-order guaranteed cost consensus problems were discussed in [39,40]. It is found that most of existing works [27–33,35,37–39] studied the guaranteed consensus based on the undirected communication topology, whether the topology can be extended to the directed topology is one of our main motivations of this paper.

It is worth noting that the inertias of agents are not take into consideration in [27–40]. In practical applications, the inertias should be considered in the second-order systems. For instance, in the attitude control of rigid bodes, the control input is the torque on each axis rather than angular acceleration. As shown in [41–43], the heterogeneity of agents' inertias over a directed topology may lead to unstable group behaviors. Therefore, they can not be ignored.

In this paper, the guaranteed cost consensus for second-order multi-agent systems with heterogeneous inertias is studied. We make the following specific contributions. First, the absolute and relative velocity damping protocols are designed for second-order multi-agent systems. Based on Lyapunov stability theory, some sufficient criteria are obtained for achieving guaranteed cost consensus. Second, by introducing two kinds of auxiliary variables, the guaranteed cost consensus problem of second-order multi-agent system with directed topology can be well transformed into the stabilization problem, which generalizes the existing works [37,39] where guaranteed cost consensus were considered via the state-space decomposition approach. Third, the communication input time delay is considered and two kinds of distributed protocols with the input time delay are designed. By constructing appropriate Lyapunov–Krasovskii functional, some sufficient conditions are derived for reaching guaranteed cost consensus. In addition, the upper bounds of the cost functions are obtained. It is found that the guaranteed cost J^* depends on the initial conditions of the Lyapunov function or Lyapunov–Krasovskii functional.

The rest of this paper is organized as follows. In Section 2, some preliminaries in algebraic graph theory, lemmas, definition and problem description are provided. In Section 3, some sufficient criteria are presented for reaching the guaranteed cost consensus under the absolute velocity damping protocol. In Section 4, the relative velocity damping protocols with and without the input time delay are proposed and some sufficient conditions for achieving guaranteed cost consensus are obtained. In Section 5, three examples are presented to show the effectiveness of the theoretical results. A short conclusion is given in Section 6.

Notations. In this paper, let $R^{N\times N}$ be the set of real matrices. R^n denotes the n-dimensional Euclidean space. Let I_N be the identity matrix with dimension N, and $\mathbf{1}_N$ be the N-dimensional column vector with each entry being 1. $\|\cdot\|$ represents the Euclidean norm. For a square matrix A, A^T represents its transpose, $\lambda_{\min}(A)$ and $\lambda_{\max}(A)$ are the minimum eigenvalue and maximum eigenvalue of A, respectively, $\|A\| = \sqrt{\lambda_{\max}(A^TA)}$. For a real symmetric matrix B, $B>0$ $(B<0)$ if B is positive (negative) definite, and $*$ represents its symmetrical part. diag$(\cdot)$ represents the diagonal matrix.

2. Preliminaries and problem description

2.1. Graph theory

A weighted digraph with N nodes is described by $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ where $\mathcal{V} = \{v_1, \dots, v_N\}$, and $\mathcal{E} \subseteq \{(v_i, v_j) : v_i, v_j \in \mathcal{V}, i \neq j\}$ are the node set and the edge set, respectively. A directed edge e_{ij} in the graph $\mathcal{G}$ is denoted by an order pair of nodes (v_i, v_j). The matrix $\mathcal{A} = [a_{ij}]_{N\times N}$ denotes the weighted adjacency matrix with $a_{ij} \geq 0$ and $a_{ii} = 0$, in which $a_{ij} > 0$ if and