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Shape-preserving piecewise rational interpolation with higher order continuity

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ABSTRACT

A united form of the classical Hermite interpolation and shape-preserving interpolation is presented in this paper. The presented interpolation method provides higher order continuous shape-preserving interpolation splines. The given interpolants are explicit piecewise rational expressions without solving a linear or nonlinear system of consistency equations. By setting parameter values, the interpolation curve can be generated by choosing the classical piecewise Hermite interpolation polynomials or the presented piecewise rational expressions. For monotonicity-preserving and convexity-preserving interpolation, the appropriate values of a parameter are given on each subinterval. Numerical examples indicate that the given method produces visually pleasing curves.

1. Introduction

In scientific data visualization and industrial design, we often require to generate a smooth function that interpolates a prescribed set of data. The interpolation function is often needed to preserve certain geometric shape properties of the data such as monotonicity or convexity. During the past 40 years, various shape-preserving interpolation methods have been proposed.

Many polynomial spline methods have a common feature in that no additional knots need to be supplied, see [1,11,35]. High smoothness interpolation was discussed in [13]. Cubic spline interpolations were analyzed in [14,30]. The methods of preserving monotonicity were presented in [32,39]. Monotone and convex spline interpolations were considered in [43]. In contrast, the papers [22,50] discussed the methods by adding one or two additional knots on the subinterval so that the monotonicity or convexity of the data is preserved. Shape preserving C^2 cubic interpolations were discussed in [22,44]. In [50], it was mentioned that the user could interactively adjust the slopes and knot locations in order to alter the shape of the interpolating curves as desired.

Rational interpolant can produce satisfying shape-preserving interpolation. There are many effective methods for the construction of shape-preserving interpolants. In [2,33,47], rational cubic interpolation splines were constructed to visualize positive data. In [15,25], rational quadratic interpolations were discussed for monotonicity-preserving. Rational cubic interpolation in [1,45,46] are appropriate for monotonicity-preserving. Rational cubic interpolation in [9,10] are appropriate for convexity-preserving.

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The recursive construction of Hermite spline interpolation was presented in [28]. By choosing the derivative values at knots, shape-preserving interpolants can be discussed. Shape preserving Hermite interpolations were considered in [40,42]. The subdivision algorithms were presented for monotone and convex Hermite interpolants in [41].

Relative to construct C^1 shape-preserving interpolants, it is a more difficult task to construct a C^2 shape-preserving interpolant, see [16,18,36]. Shape preserving C^2 cubic and quintic interpolation were discussed in [21,44]. For C^2 continuity, the solution of the consistency equations was concerned, see [9,16,18].

For shape-preserving approximation, some error estimates have been discussed in [3,4,8]. Monotone approximation were presented in [19,20]. As we know, the approximation by B-splines is shape-preserving, see [6], but the approximation order is not satisfied. In [5], a simple convexity-preserving algorithm was given by B-splines.

Recently, in application to computer-aided design, the variable degree polynomial spline was presented in [12]. Interpolation methods with tension control were given in [24,26]. Shape-preserving curve representations were considered in [31,48]. Shape-preserving L_1 splines were discussed in [37,38].

For C^2 continuity, when the methods are concerned to solve the consistency equations, shape-preserving properties will lead to some constraint conditions on the interpolation data and changes to any data will require solving again all the equations. Therefore, it is difficult to achieve monotonicity-preserving and convexity-preserving at the same time. In [15], for strict monotone data, the existence and uniqueness of a positive solution of the non-linear equations were proved. In [17], for strict convex data, the existence and uniqueness of a solution of the non-linear equations satisfying the convexity constraint were shown. In [18], it was mentioned that the proper choice of the parameter to guarantee shape preservation is still an unsettled question.

There are few C^2 continuous interpolation methods which are appropriate not only for monotonicity-preserving but also for convexity-preserving. In [15,39], the convexity-preserving properties of the interpolants were not discussed. In [7,10,17,49], the monotonicity-preserving properties of the interpolants were not discussed. In the paper [27], the given interpolant is convexity-preserving and C^2 continuous without solving a global system of equations, and the approximation order is 3. With second order precision in general, the given methods in [29,51,52] were C^2 continuous interpolation methods which are appropriate not only for monotonicity-preserving but also for convexity-preserving.

The first motivation of this paper is to present higher order continuous piecewise interpolants which are appropriate not only for monotonicity-preserving but also for convexity-preserving. We will construct higher order continuous interpolants without solving a global system of consistency equations or adding additional knots on the subinterval.

The second motivation of this paper is to present shape-preserving interpolants which are the generalization of the classical Hermite interpolants. The classical Hermite interpolants are of higher order approximation. The classical Hermite interpolants may be shape-preserving for some data and may not be shape-preserving for other data. Based on the properties of the given data, we can choose interpolants on each subinterval.

The rest sections of this paper are organized as follows. In next section, the piecewise expressions of the rational interpolants are presented and some properties of the interpolants are described. The shape-preserving properties are discussed in Section 3. Some numerical examples and conclusions are given in Sections 4 and 5, respectively.

2. Piecewise rational interpolants

Let $a = x_1 < x_2 < \cdots < x_n = b$ be a partition of the interval $[a, b]$ and $f_i^{(k)}$ be given values corresponding to knots x_i, $f_i = f_i^{(0)}$, $i = 1, 2, \ldots, n$, $k = 0, 1, \ldots, m$, then we put $h_i = x_{i+1} - x_i$, $\Delta f_i = (f_{i+1} - f_i)/h_i$ for $i = 1, 2, \ldots, n-1$.

For $x \in [x_i, x_{i+1}]$, $i = 1, 2, \ldots, n-1$, we will construct the interpolants

$$H_{i,m}(x) = L_i(x) + \sum_{j=1}^{m} r_{i,j}(x) h_i^j, \tag{1}$$

where m is a positive integer, $L_i(x) = (1-t)f_i + t f_{i+1}$, $r_{i,j}(x) = p_{i,j}(x)/q_{i,j}(x)$,

$$p_{i,j}(x) = (1-t)^{j+1} t^j \alpha_{i,j} + (1-t)^j t^{j+1} \beta_{i,j}, \quad q_{i,j}(x) = 1 + (1-t)t w_{i,j},$$

and $t = (x - x_i)/h_i$. Thus, for $x \in [a, b]$, we obtain piecewise interpolant

$$H_m(x) = H_{i,m}(x), \quad L(x) = L_i(x), \quad x \in [x_i, x_{i+1}],\ i = 1, 2, \ldots, n-1,$$

where $L(x)$ is a piecewise linear interpolant.

The parameters $\alpha_{i,j}$ and $\beta_{i,j}$ will be chosen so that the higher derivatives of the interpolants satisfy

$$H_{i,m}^{(k)}(x_i) = f_i^{(k)}, \quad H_{i,m}^{(k)}(x_{i+1}) = f_{i+1}^{(k)}, \quad k = 1, 2, \ldots, m. \tag{2}$$

Thus, the interpolants H_m are C^m continuous piecewise interpolants.

Obviously, the interpolants H_m are the classical Hermite interpolants when all $w_{i,j} = 0$. we restrict $q_{i,j}(x)$ to be a quadratic polynomial so that the constructed interpolant is simple and shape-preservation properties can be discussed conveniently. We have $q_{i,j}(x) > 0$ when $w_{i,j} > -4$. When $w_{i,j} \ge 0$, we have $q_{i,j}(x) \ge 1$. By taking large values of $w_{i,j}$, we expect that the piecewise interpolant H_m approximates the piecewise linear interpolant $L(x)$ well. Therefore, we would like to take $w_{i,j} \ge 0$ to achieve satisfying shape of the interpolation curves.