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Stochastic stability for distributed delay neural networks via augmented Lyapunov–Krasovskii functionals☆

Yonggang Chen [a], Zidong Wang [b,c,*], Yurong Liu [d], Fuad E. Alsaadi [e]

[a] School of Mathematical Sciences, Henan Institute of Science and Technology, Xinxiang 453003, China
[b] College of Electrical Engineering and Automation, Shandong University of Science and Technology, Qingdao 266590, China
[c] Department of Computer Science, Brunel University London, Uxbridge, Middlesex UB8 3PH, United Kingdom
[d] Department of Mathematics, Yangzhou University, Yangzhou 225002, China
[e] Faculty of Engineering, King Abdulaziz University, Jeddah 21589, Saudi Arabia

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ABSTRACT

This paper is concerned with the analysis problem for the globally asymptotic stability of a class of stochastic neural networks with finite or infinite distributed delays. By using the delay decomposition idea, a novel augmented Lyapunov–Krasovskii functional containing double and triple integral terms is constructed, based on which and in combination with the Jensen integral inequalities, a less conservative stability condition is established for stochastic neural networks with infinite distributed delay by means of linear matrix inequalities. As for stochastic neural networks with finite distributed delay, the Wirtinger-based integral inequality is further introduced, together with the augmented Lyapunov–Krasovskii functional, to obtain a more effective stability condition. Finally, several numerical examples demonstrate that our proposed conditions improve typical existing ones.

1. Introduction

Over the past several decades, the delayed neural network has gained considerable research attention due to its clear application insights in a variety of research areas such as signal and image processing, pattern recognition and optimization, see e.g. [8,20,25,33,42]. Time delays are inevitable in neural networks since the communications/transmissions between neurons cannot be instantaneous. The motivation for studying the delayed neural networks is mainly twofold: the first is the widely acknowledged successes of neural network applications in many important fields, and the second is the possibly undesirable dynamic behaviors caused by time delays in neural networks [1,9]. So far, three main types of time delays have been thoroughly examined in the context of recurrent neural networks, namely, the discrete delay [1,9], the neutral delay [6,12], and the distributed delay [30,37]. In addition, it has been revealed that the synaptic transmission in real nervous systems should be a noisy process and certain noisy inputs could destabilize a neural network [23,31]. As such, it is of practical significance to investigate the dynamical behaviors of neural networks with both time delays and stochastic perturbations.

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* Corresponding author.
E-mail address: zidong.wang@brunel.ac.uk (Z. Wang).

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The stability has proven to be one of the most important dynamical behaviors and, up to now, there has been a rich body of literature on the stability analysis problems for delayed neural networks (with or without stochastic perturbations), where the majority of the results have been obtained based on algebraic inequality and M-matrices. In particular, it is pretty convenient to verify the stability criteria expressed by linear matrix inequalities (LMIs) through available software package. In the past few years, a number of advanced techniques have been developed to derive less conservative LMI-based stability conditions [2,3,11,13,18,28,32,36,40,41]. For example, the integral inequality approach has been proposed in [40], a general delay decomposition approach has been developed in [3] and an augmented Lyapunov–Krasovskii (L–K) functional has been constructed in [13] to reduce the possible conservatism of the obtained asymptotic stability conditions. In [11,28], free-matrix-based integral inequalities have been proposed to establish some effective conditions for globally exponential stability.

In spite of the intensive investigation on delayed neural networks, it is worth mentioning that the majority of published results have been concerned with neural networks subject to *discrete* delays. Note that a neural network usually has a spatial nature due to the presence of an amount of parallel pathways of a variety of axon sizes and lengths [15,29,30,37] and, therefore, it is often desirable to examine the impact from the distributed delay (either finite or infinite) on the stability of neural networks [2,7,14,17,21,22,24,31,35,38,39]. Recently, in [7,35], some improved stability conditions have been proposed for neural networks with *finite* distributed delay by using the delay decomposition approach and augmented L–K functionals. Nevertheless, there still appears some room to further reduce the conservatism of the main results in [7,35] through the following ways: (1) instead of the rather conservative Jensen integral inequalities, some more up-to-date inequalities can be employed to estimate the upper bounds of the derivatives of L–K functionals, and (2) the relationships between the neuron state, the activation function and the distributed delay could be more sufficiently utilized. Similarly, most existing results concerning the infinite distributed delay (see e.g. [17]) could be further improved by adopting more dedicated L–K functionals and inequalities.

Motivated by the above discussions, the main purpose of this paper is to establish some less conservative stability conditions for stochastic neural networks with finite or infinite distributed delay. Different from the existing techniques, the augmented L–K functional approach and the delay decomposition approach are utilized to consider the stability problem of stochastic neural networks with *infinite* distributed delay. As for stochastic neural networks with *finite* distributed delay, the Wirtinger-based inequality is further incorporated to obtain effective stability conditions. Finally, several examples are given to demonstrate the reduced conservatism of the proposed conditions. The main contributions of this paper are highlighted as follows: (1) Based on the delay decomposition approach, a novel augmented L–K functional (containing double and triple integral terms) is constructed and, using such an L–K functional, an improved stability condition is obtained for stochastic neural networks with infinite distributed delay. (2) By using a combination of the augmented L–K functional, the delay decomposition idea and the Wirtinger-based inequality, an effective stability condition is established for stochastic neural networks with finite distributed delay.

Notation. The superscript "T" stands for the transpose of a matrix. $\mathbb{R}^n$ and $\mathbb{R}^{n\times n}$ refer to the n-dimensional Euclidean space and set of all $n\times n$ real matrices, respectively. $\|\cdot\|$ denotes the Euclidean norm of a vector. $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathcal{P})$ is a complete probability space with a filtration $\{\mathcal{F}_t\}_{t\geq 0}$ satisfying the usual conditions. $P>0$ denotes that P is a real, symmetric and positive definite matrix. $\mathbb{E}\{\cdot\}$ stands for the mathematical expectation operator. Sym$\{N\}$ is the shorthand notation for the matrix $N+N^T$, and I denotes an identity matrix with proper dimension. Matrices, if not explicitly stated, are assumed to have compatible dimensions.

2. Problem formulation

Consider the following stochastic neural network with infinite distributed delay:

$$\mathrm{d}x(t) = \left[-Ax(t) + Bf(x(t)) + C\int_{-\infty}^{t} k(t-s)f(x(s))\,\mathrm{d}s\right]\mathrm{d}t + g(t, x(t))\mathrm{d}\omega(t) \tag{1}$$

where $x(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^T \in \mathbb{R}^n$ is the neuron state, $f(x(t)) = [f_1(x_1(t)), f_2(x_2(t)), \ldots, f_n(x_n(t))]^T \in \mathbb{R}^n$ represents the neuron activation function, $A = \mathrm{diag}\{a_1, a_2, \ldots, a_n\}$ is a diagonal matrix with $a_i > 0$ $(i = 1, 2, \ldots, n)$, B and C are the connection weight matrix and the delayed connection weight matrix, respectively, $\omega(t)$ is an m-dimensional Brownian Motion defined on $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathcal{P})$.

As for the stochastic neural network with a finite distributed delay $h>0$, by modifying the model (1) with $k(t)\equiv 1$ $(t\geq 0)$, we have the following neural network model [31]:

$$\mathrm{d}x(t) = \left[-Ax(t) + Bf(x(t)) + C\int_{t-h}^{t} f(x(s))\,\mathrm{d}s\right]\mathrm{d}t + g(t, x(t))\mathrm{d}\omega(t). \tag{2}$$

For models (1)–(2), we make the following assumptions.

Assumption 1. The activation functions $f_i(x_i)$ $(i = 1, 2, \ldots, n)$ are continuously differentiable with $f_i(0) = 0$ and satisfy

$$\sigma_i^- \leq \frac{f_i(x_i)}{x_i} \leq \sigma_i^+, \ \forall\, x_i \neq 0, \ i = 1, 2, \ldots, n$$